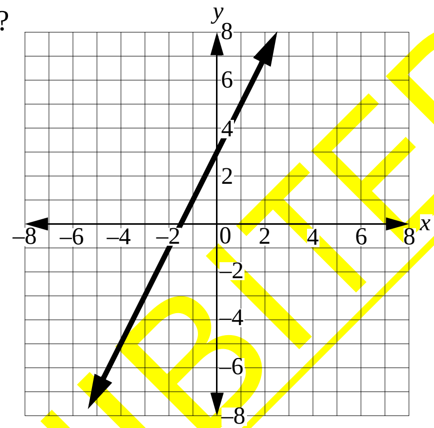
Section 9.4
Exponential Functions (Growth and Decay)

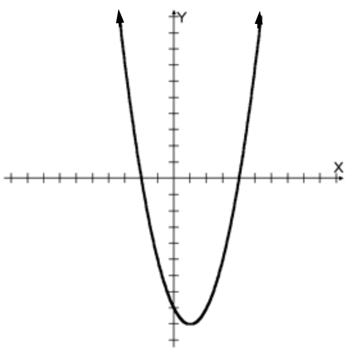
What shape would a line on a graph have if the exponent is a variable?

From past experience we know that an equation such as

$$y = 2x + 3$$

is a straight line (see graph at right).





Also from past experiences we discovered that squaring the base "x", will turn the line into a curve,

$$y = x^2 - 2x - 8$$

specifically a parabola (see graph to the left).

What shape would a line on a graph have if the exponent is a variable?

The shape is a curve that increases (climbs) rapidly. Naturally, we call this type of curve "exponential growth." The particular equation of the graph in figure 1 is:

$$y = 2^x$$

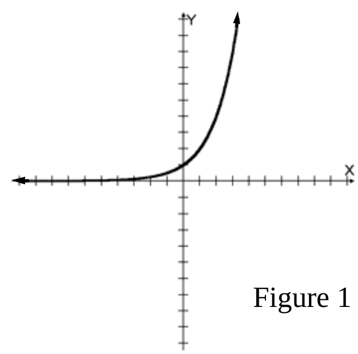


Figure 1

VARIATIONS OF THE EXPONENTIAL CURVE

Because the coefficient is negative, a curve such as $y = -2^x$ (figure 2)

will send the curve the opposite way, towards the negative side.

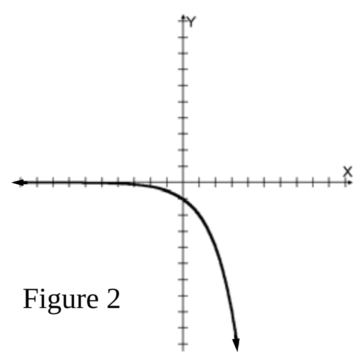


Figure 2

A decimal base such as $y = 0.5^x$

will make the curve decrease (drop) rapidly. This type of curve is referred to as "exponential decay." See figure 3.

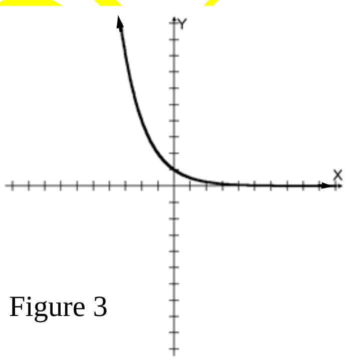


Figure 3

Example in exponential growth:

On her birthday, a five-year-old receives a $5,000 gift and her parents place the money in a bank account that earns interest of 7% per year (7 cents for very dollar deposited). How much money would be available in the account when the girl turns 20 (15 years later)?

After the first year one dollar would have grown to $1.07, and after the second year the $1.07 would have to grow to $1.1449 ($1.07 \times 1.07$), and after the third year to $1.225, and so on.

This problem fits the exponential growth pattern because the more interest is accumulated, the higher the interest earned. In other words, the money grows rapidly by compounding the interest amount. The exponential equation would be:

$$y = a(1 + i)^x$$

Where y is the final amount
a is the initial amount
i is the rate (percent) of increase per period in decimal form
x is the number of periods (years)

Solving for y $\quad y = 5000\,(1 + 0.07)^{15}$

Because multiplying 1.07 to the 15th power is tedious, we use the y^x function of a scientific calculator*:

Press 1.07
Press y^x
Press 15
Press =

$$(1 + 0.07)^{15} = 2.759031541$$

$$y = 5000\,(2.759031541) = 13795.16$$

The girl at twenty will have $13,795.16. She earned $8,795.16 in interest.

Example in exponential decay:

A chlorine solution decays 50% (0.5) for every week it is left uncovered. If the original amount of chlorine used is 2560 ounces (20 gallons), how much chlorine is available in the solution after 12 weeks?

If it decays 50% (half the amount) every week, after the first week there is $\frac{2560}{2}$ = 1280 ounces, after the second week there is $\frac{1280}{2}$ = 640 ounces available, and so on.

This problem fits the exponential decay pattern because the amount of chlorine drops rapidly. The exponential equation would be

$$y = a(1 - i)^x$$

Compare this equation to the one above. For growth, i is added, for decay, i is subtracted.

Where y is the final amount
a is the initial amount
i is the rate (percent) of decrease per period in decimal form
x is the number of periods (weeks)

Solving for y $\quad y = 2560\,(1 - 0.5)^{12}$

Because multiplying 0.5 to the 12th power is tedious, we use the y^x function of a scientific calculator*:

Press 0.5
Press y^x
Press 12
Press =

$$(1 - 0.5)^{12} = (0.5)^{12} = 0.000244141$$

$$y = 2560\,(0.000244141) = 0.625$$

The amount of available chlorine after 12 weeks is 0.625 ounces (slightly more than a tablespoon).

*Performed on a TI-35X calculator.

Practice:
Solve and find the value of y when x is 9. Round off answers to the nearest thousandth.

1. $y = 520\,(1 - 0.5)^x$
2. $y = 2300\,(0.76)^x$
3. $y = 0.95\,(1 + 0.4)^x$
4. $y = 85600\,(1 - 0.05)^x$
5. $y = 95000\,(0.15)^x$
6. $y = 0.56\,(1 - 0.33)^x$
7. $y = 756\,(1 + 0.06)^x$
8. $y = 45000\,(0.55)^x$
9. $y = 3400\,(1 + 0.99)^x$
10. $y = 1700\,(0.07)^x$
11. $y = 65000\,(1 - 0.01)^x$
12. $y = 0.65\,(1 + 0.65)^x$
13. $y = 65000\,(0.055)^x$
14. $y = 54\,(1 - 0.0045)^x$
15. $y = 840\,(1 + 0.24)^x$
16. $y = 5700\,(0.35)^x$
17. $y = 820\,(0.077)^x$
18. $y = 355\,(1 + 0.8)^x$
19. $y = 8200\,(0.33)^x$
20. $y = 380\,(1 - 0.035)^x$
21. $y = 1200\,(1 + 0.028)^x$
22. $y = 0.24\,(0.6)^x$
23. $y = 93\,(1 - 0.555)^x$
24. $y = 8540\,(1 + 0.074)^x$

Solve

25. The year she retires, Gerri receives a severance check for $135,000. If she leaves this amount in a 6.25% retirement account for 10 years, how much money would be available in the account after ten years? How much interest has accumulated?
26. A large factory has 25,000 bulbs. If 4% of the bulbs fail every week, how many original bulbs remain after 26 weeks?
27. The population of a certain city is projected to decrease 2% per year. If the city has approximately 1.2 million inhabitants today, at this rate, what could be the estimated population 15 years into the future. Round off to the nearest thousand?
28. A certain chemical evaporates at the rate of 0.5% per hour. If the container holds 2 liters of the substance, how long before the amount left in the container reaches 1 liter?
29. A woman born on January 1, 1946, is ready to retire on December 31, 2006. If her parents placed $500 in a bank the day she was born and the bank pays 6.5% per year, how much money will she have in the account when she retires?
30. The population of a country in Central America grows at the rate of 2.4% per year. If the present population is 8 million, how many people could they expect in 10 years?