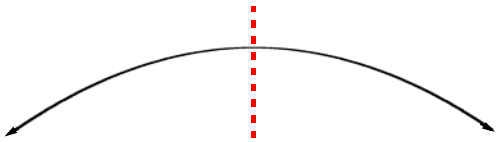
Section 9.1
Solving Quadratic Equations by Factoring

Similar to a *linear equation* ($y = mx + b$), a *quadratic equation* has two variables (x, y), but unlike in a linear equation, one of the variables is a square: $\boldsymbol{y = ax^2 + bx + c}$. *(quadratic = square)*

As the name indicates, a linear equation, when plotted, forms a line. On the other hand, a quadratic equation, when plotted, forms a curve.

A quadratic equation doesn't form just any curve, but a curve that is symmetrical (half of the curve is the reflection of the other half).

Called a parabola, it is the shape of the path of a baseball (or projectile) in flight.



The solution of a quadratic is achieved through factoring, if we remove the equal sign (=) and the y variable:

$y = ax^2 + bx + c \longrightarrow ax^2 + bx + c$

This is the type of trinomial studied in section 7.5. For example,

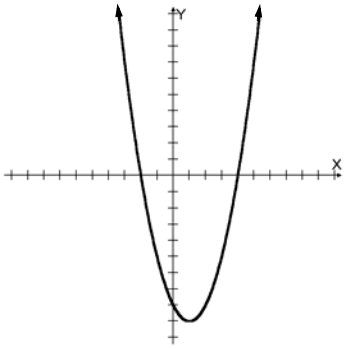
$x^2 - 2x - 8$ can be factored into $(x - 4)(x + 2)$

putting back the equal sign and the y variable it becomes: $y = (x - 4)(x + 2)$

This is the type of equation to be solved in this section.

HOW TO SOLVE A QUADRATIC EQUATION

Because these equations represent curves and curves come around (unlike the straight line of linear equations that crosses axes only once), in many cases a curve must cross the same axis twice. The figure to the right shows this.



A parabolic curve always comes back—like a boomerang. Notice how in the graph it comes down from left to right and crosses the x-axis twice before going off to the top right. Specifically, notice how the curve comes down and crosses the x-axis at –2, and comes back up crossing the x-axis again at +4. Factoring a quadratic equation, it is these very points we seek. These points become the "roots" or "zeros" of the quadratic.

The equation plotted in the previous graph is $y = x^2 - 2x - 8$

Because the value of y is zero ($y = 0$) when the parabola crosses the x-axis, we now set up the equation equal to zero to find the specific values of x when the curve crosses the x-axis (roots).

$0 = x^2 - 2x - 8 \longrightarrow$ Factoring the trinomial $\longrightarrow (x - 4)(x + 2) = 0$

Because it is not known which of the two binomials, $(x - 4)$ or $(x + 2)$, is equal to zero, setting up each binomial equal to zero is the way to find both values for x:

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ $\qquad$ $x = -2$

These answers are called the roots of the quadratic and can be verified in the graph above.

Example: Solve $(x + 2)(x – 3) = 0$

$(x + 2) = 0$ or $(x – 3) = 0$

$x = –2$ $\quad$ $x = 3$

Example: Solve $\left(\frac{1}{3}x – 2\right)\left(\frac{2}{5}x + 3\right) = 0$

$\frac{1}{3}x – 2 = 0$ or $\frac{2}{5}x + 3 = 0$

$\frac{1}{3}x = 2$ $\quad$ $\frac{2}{5}x = –3$

$x = 6$ $\quad$ $x = \frac{–15}{2} = –7.5$

Example: Solve $0 = x^2 + 7x + 12$

$0 = x^2 + 7x + 12$ (factor trinomial)

$0 = (x + 3)(x + 4)$

$x + 3 = 0$ $\quad$ $x + 4 = 0$

$x = –3$ $\quad$ $x = –4$

Example: Solve $x^2 – 5x – 4 = 2$

$x^2 – 5x – 4 = 2$ (move 2 to left-side and add to –4)

$x^2 – 5x – 6 = 0$ (factor)

$(x – 6)(x + 1) = 0$

$x – 6 = 0$ $\quad$ $x + 1 = 0$

$x = 6$ $\quad$ $x = –1$

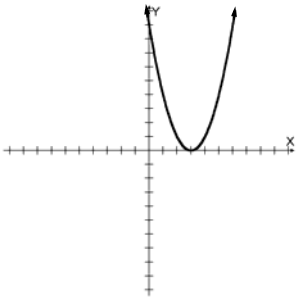
Example: Solve $x^2 – 6x = –9$

$x^2 – 6x + 9 = 0$ (perfect trinomial square)

$(x – 3)(x – 3) = 0$

$x – 3 = 0$ $\quad$ $x – 3 = 0$

$x = 3$ $\quad$ $x = 3$



One double, equal root, 3. This curve comes down and "touches" the x-axis only at 3, instead of crossing it.

Example: Solve $x^2 = 6x$

$x^2 – 6x = 0$ (find common factor)

$x(x – 6) = 0$

$x = 0$ $\quad$ $x – 6 = 0$

$x = 6$

Example: Solve $2x^2 + 9x + 10 = 0$

$2x^2 + 9x + 10 = 0$

$(2x + 5)(x + 2) = 0$

$2x + 5 = 0$ $\quad$ $x + 2 = 0$

$2x = –5$ $\quad$ $x = –2$

$x = –\frac{5}{2} = –2.5$

Example: Solve $6x^2 + x = 12$

$$6x^2 + x - 12 = 0$$
$$(2x + 3)(3x - 4) = 0$$

$2x + 3 = 0$ | $3x - 4 = 0$

$2x = -3$ | $3x = 4$

$x = -\frac{3}{2} = 1.5$ | $x = \frac{4}{3} = 1.\overline{3}$

Practice:
Find the roots of the quadratic.

1. $(x - 3)(x + 4) = 0$
2. $(x + 1)(x + 5) = 0$
3. $(x - 2)(x + 7) = 0$
4. $(x - 0.2)(x - 4.5) = 0$
5. $(x - \frac{1}{4})(x + 4) = 0$
6. $(x - 3)(x - \frac{2}{3}) = 0$
7. $(2x + 9)(3x + 8) = 0$
8. $(6x - 5)(x - 7) = 0$
9. $(3x - 1)(2x + 5) = 0$
10. $(3x + 8)(x + 10) = 0$
11. $(4x - 5)(5x + 7) = 0$
12. $(x - 12)(\frac{3}{4}x - 6) = 0$
13. $(\frac{2}{3}x - 8)(3x - 5) = 0$
14. $x^2 + 5x + 4 = 0$
15. $x^2 + 7x + 12 = 0$
16. $a^2 = 25$
17. $x^2 + 5x - 14 = 0$
18. $y^2 + 8y + 16 = 0$
19. $x^2 - 3x + 2 = 0$
20. $y^2 = 9$
21. $y^2 + 14y + 49 = 0$
22. $x^2 - 7x - 18 = 0$
23. $x^2 + 10x = -24$
24. $g^2 - 2g = 15$
25. $0 = y^2 - 13y + 22$
26. $x^2 - 10 = -9x$
27. $w^2 - 16w = 36$
28. $y^2 - 7y = 44$
29. $c^2 + 20 = 12c$
30. $x^2 + 3x - 28 = 0$
31. $n^2 - 9n - 22 = 0$
32. $64y^2 + 80y + 25 = 0$
33. $x^2 - 4 = 0$
34. $3x^2 + x + 10 = 0$
35. $2x^2 + 9x - 5 = 0$
36. $4a^2 + 15a + 14 = 0$
37. $2x^2 + 7x - 15 = 0$
38. $2x^2 + 11x + 12 = 0$
39. $3a^2 + 10a + 8 = 0$
40. $5x^2 + 3x - 14 = 0$
41. $6x^2 + 23x + 20 = 0$
42. $7p^2 + 25p - 12 = 0$
43. $7c^2 + 8c + 1 = 0$
44. $2x^2 + 15x - 8 = 0$
45. $3r^2 + 24r - 27 = 0$
46. $5x^2 + 6x = 8$
47. $4t^2 + 10t + 6 = 0$
48. $5x^2 - x = 22$
49. $2x^2 + 6x - 8 = 0$
50. $6y^2 - 11y = -3$
51. $8x^2 - 22x = -15$
52. $7y^2 - 15y - 18 = 0$
53. $10x^2 + 12x = 16$
54. $3m^2 - 15m = -12$
55. $3x^2 - 16x = -21$
56. $5n^2 - n - 22 = 0$
57. $7d^2 - 2d = 32$
58. $28x^2 - 25x = 42$
59. $25z^2 - 9z = 16$
60. $24x^2 - 6x = 18$
61. $32u^2 - 20u - 12 = 0$
62. $3j^2 + 81 = 36j$
63. $0 = 7k^2 + 21k - 28$
64. $8x^2 + 16x - 24 = 0$
65. $10x^2 - 20x = 30$
66. $12y^2 - 20y = 48$
67. $32x^2 + 56x - 36 = 0$
68. $24g^2 - 9g = 15$