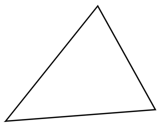
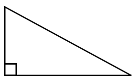
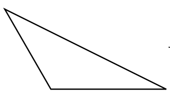
Section 8.4
The Pythagorean Theorem

Over four thousands years ago somewhere in China, or perhaps India, some say Egypt, a most remarkable tool for solving problems surfaced. Called *the pythagorean theorem*, after the 5th Century BCE Greek mathematician who proved it, it is a method of using a particular property of the right triangle to find the length of a missing side when the two other sides are known.

The angles of a triangle—a three-sided polygon—classify triangles as

ACUTE  when all angles are less than 90°

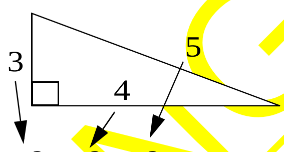
RIGHT  when one angle is 90°

OBTUSE  when one angle is greater than 90°

The Pythagorean Theorem concerns the right triangle (one angle 90°) only.

This theorem says that if we square all the sides of a triangle (this is important: ONLY when they are squared), the largest of the sides (the largest side is the one opposite the 90° angle) is equal to the sum of the other two sides.

Example:



$3^2 + 4^2 = 5^2$

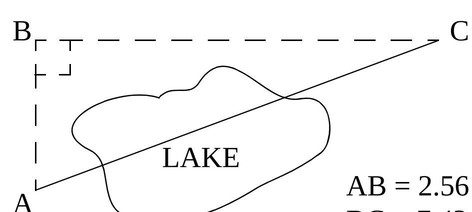
$9 + 16 = 25$

In this example, 3 has been squared, so has 4 and 5. If we add before squaring them, they don't "add up." Once squared, however, **3**, as **9**, and **4** as **16**, the sum is **25**, which is **5** squared.

The pythagorean theorem is very valuable, and it can be used, for example, to compute distances that can't be physically measured or for checking their accuracy.

Example: What is distance $\overline{AC}$ across the lake?

Measuring $\overline{AB}$ and $\overline{BC}$ was achieved over land; however, there is no need to measure $\overline{AC}$, for it can be computed using the pythagorean theorem.



AB = 2.56 miles
BC = 7.43 miles

Square $\overline{AB}$ $(2.56)^2 = 6.5536$ Square $\overline{BC}$ $(7.43)^2 = 55.2049$

Add $\overline{AB}$ and $\overline{BC}$ $6.5536 + 55.2049 = 61.7585$

Find the square root $\sqrt{61.7585} = 7.859$

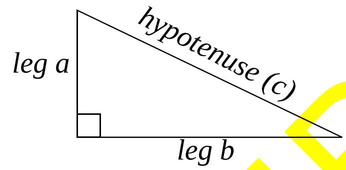
The distance across the lake is **7.859 miles** (rounded off)

THE PYTHAGOREAN FORMULA

The pythagorean theorem can be stated using the following formulas that compute each side:

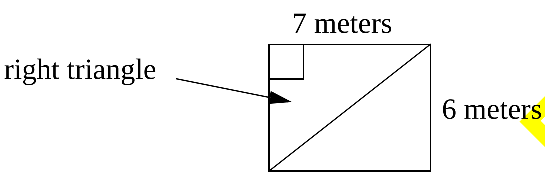
$$c = \sqrt{a^2 + b^2} \qquad a = \sqrt{c^2 - b^2} \qquad b = \sqrt{c^2 - a^2}$$

Called the *hypotenuse*, we'll make the largest side "***c***"
The other two sides we'll call leg "***a***" and leg "***b***."



Example: A carpenter is building the frame of a room. If the room is 6 meters wide and 7 meters long, how can she check if the room is accurately rectangular?

In a rectangle, the diagonals are equal. The best way for the carpenter to know if she has a rectangle with 90° angles, is to make the diagonals equal. What is the length of the diagonals if the sides are 6 and 7?

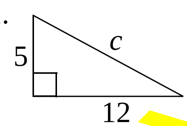


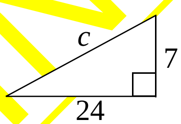
$c = \sqrt{7^2 + 6^2} = \sqrt{49 + 36} = \sqrt{85} = 9.22$

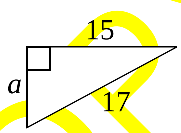
The diagonals must be **9.22 meters** for the room to be a rectangle.

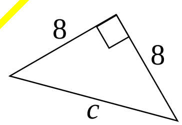
Practice:

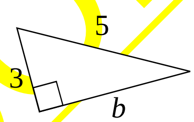
Find the missing side.

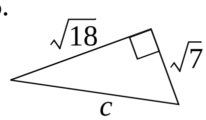
1. 

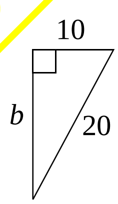
2. 

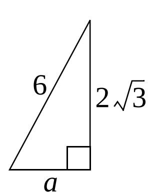
3. 

4. 

5. 

6. 

7. 

8. 

9. $a = 6, b = 8$
10. $a = 16, b = 18$
11. $a = 7, b = 11$
12. $a = 5, c = 20$
13. $a = 4, b = 10$
14. $a = \sqrt{20}, b = 8$
15. $b = 3, c = 6$
16. $a = 9, b = \sqrt{12}$
17. $a = 5, c = 20$
18. $b = 3\sqrt{5}, c = 4\sqrt{6}$
19. $a = \sqrt{8}, b = 14$
20. $a = 7, c = 13$
21. $a = \sqrt{7}, b = \sqrt{7}$
22. $a = 40, c = 40$
23. $b = \sqrt{15}, c = 8$
24. $a = 18, b = 18$
25. $a = 11, c = 22$
26. $b = \sqrt{14}, c = 9$
27. $a = 17, b = 200$
28. $a = 7, c = 25$
29. $b = 5, c = \sqrt{70}$
30. $a = 8, b = 15$
31. $a = \sqrt{1}, b = \sqrt{1}$
32. $b = 12, c = 13$