

## Section 8.3

# Radical Equations

Similarly to the way any equation is solved, an equation with radicals is solved by using the inverse property. In this case, the inverse of a radical is found by raising the radical to a like power. Therefore, the inverse of the **square root** is the **square**.

**Example:** Solve  $\sqrt{x} = 5$

square both sides  $(\sqrt{x})^2 = 5^2$   
 $x = 25$

**Example:** Solve  $\sqrt{3x-5} = 13$

move  $-5$   $\sqrt{3x-5} + 5 = 13 + 5$   
 $\sqrt{3x} = 18$

Always isolate radical to the left before squaring.

square both sides  $(\sqrt{3x})^2 = 18^2$   
 divide by 3  $3x = 324$   
 $x = 108$

**Example:** Solve  $\sqrt{3x-5} = \sqrt{2x+8}$

square both sides  $(\sqrt{3x-5})^2 = (\sqrt{2x+8})^2$   
 $3x-5 = 2x+8$   
 solve for  $x$   $3x-2x = 5+8$   
 $x = 13$

**Example:** Solve  $3\sqrt{x-4} = 5\sqrt{6-x}$

square both sides  $(3\sqrt{x-4})^2 = (5\sqrt{6-x})^2$   
 $9(x-4) = 25(6-x)$   
 $9x-36 = 150-25x$   
 $9x+25x = 150+36$   
 $34x = 186$   
 $x = 5.47$  (rounding to the nearest hundredth)

**Example:** Solve  $\sqrt{3x^2+24x-18} = 3$

square both sides  $(\sqrt{3x^2+24x-18})^2 = 3^2$

subtract 9 from both sides  $3x^2+24x-18 = 9$

divide equation by common factor 3  $3x^2+24x-27 = 0 \longrightarrow x^2+8x-9 = 0$

Equation is now factored: Which two integers give a product of  $-9$  and a difference of  $+8$ ?

Only two possible multiplications yield a 9:  $9 \times 1$  and  $3 \times 3$ , and only  $9 - 1 = 8$

$$\begin{array}{l} \text{Factoring } (x+9)(x-1) = 0 \\ x+9 = 0 \quad x-1 = 0 \\ x = -9 \quad x = 1 \end{array}$$

### EQUATION WITH RADICALS THAT YIELD NO VALUE

Some radical equations cannot be solved.

In examples

$$\sqrt{x} = -8 \quad -\sqrt{x} = 8 \quad \sqrt{3x-5} = \sqrt{3x+8}$$

all three equations shown will be either negative roots which do not exist, or equations that cancel.

### Practice:

Solve.

1.  $\sqrt{x} = 1$
2.  $\sqrt{5x+3} = 18$
3.  $\sqrt{5x-7} = \sqrt{4x+10}$
4.  $\sqrt{x} = 2$
5.  $5\sqrt{x-1} = 4\sqrt{10-x}$
6.  $\sqrt{3x-5} = 13$
7.  $\sqrt{3x-5} = \sqrt{2x+8}$
8.  $\sqrt{x} = 3$
9.  $3\sqrt{x-4} = 5\sqrt{6-x}$
10.  $\sqrt{4x+8} = 12$
11.  $\sqrt{7x-6} = \sqrt{2x+9}$
12.  $6\sqrt{x-2} = 7\sqrt{x-5}$
13.  $\sqrt{x} = 4$
14.  $\sqrt{x-2} = 5$
15.  $\sqrt{8x-15} = \sqrt{6x-12}$
16.  $\sqrt{x} = 6$
17.  $3\sqrt{x-9} = 2\sqrt{x+4}$
18.  $\sqrt{2x+6} = 14$
19.  $\sqrt{x+37} = \sqrt{3x+29}$
20.  $\sqrt{x-8} = 4\sqrt{9-x}$
21.  $\sqrt{x} = 7$
22.  $\sqrt{6x-9} = 10$
23.  $\sqrt{9x-7} = \sqrt{4x+13}$
24.  $8\sqrt{x-5} = 2\sqrt{x-2}$
25.  $\sqrt{x} = 8$
26.  $\sqrt{7x+5} = 19$
27.  $\sqrt{10x-23} = \sqrt{9x-5}$
28.  $\sqrt{x-2} = 9\sqrt{9-x}$
29.  $\sqrt{x} = 9$
30.  $\sqrt{8x-12} = 20$
31.  $\sqrt{9x-15} = \sqrt{5x+3}$
32.  $2\sqrt{x-3} = 8\sqrt{x+5}$
33.  $\sqrt{x} = 10$
34.  $\sqrt{9x+2} = 29$
35.  $\sqrt{8x+10} = \sqrt{12x-3}$
36.  $4\sqrt{x-4} = 7\sqrt{x-4}$
37.  $\sqrt{x} = 11$
38.  $\sqrt{5x-10} = 15$
39.  $\sqrt{5x-4} = \sqrt{4x+9}$
40.  $5\sqrt{x-7} = 6\sqrt{3-x}$
41.  $\sqrt{x} = 12$
42.  $\sqrt{6x+18} = 24$
43.  $6\sqrt{x-5} = 5\sqrt{x-2}$
44.  $\sqrt{6x+25} = \sqrt{11x+7}$
45.  $\sqrt{(2x-6)^2} = 3$
46.  $\sqrt{(a-2)^2} = 6$
47.  $\sqrt{2x+9x-6} = 4$
48.  $\sqrt{(x+1)^2} = 3$
49.  $\sqrt{(13x-8)^2} = 2$
50.  $\sqrt{(5y+2)^2} = 6$
51.  $\sqrt{7x+12x+5} = 10$
52.  $\sqrt{(8t-9)^2} = 11$