

Section 8.2

Operations with Radical Expressions

MULTIPLYING RADICALS

Radicals of the same root may be multiplied under the same radical or separated under different radicals.

Example: Simplify $\sqrt{2} \times \sqrt{6}$

$$\sqrt{2} \times \sqrt{6} = \sqrt{2 \times 6} = \sqrt{12} = \sqrt{2 \times 2 \times 3} = 2\sqrt{3}$$

Example: Simplify $\sqrt{18} \times \sqrt{24}$

$$\begin{aligned}\sqrt{18 \times 24} &= \sqrt{2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 3} \\ &= 4 \times 3\sqrt{3} \\ &= 12\sqrt{3}\end{aligned}$$

Practice:

Multiply and simplify.

- $(\sqrt{7})(\sqrt{6})$
- $(\sqrt{18})(\sqrt{3})$
- $(\sqrt{5})(\sqrt{4})(\sqrt{12})$
- $(\sqrt{14})(\sqrt{10})(\sqrt{8})$
- $(\sqrt{x})(\sqrt{32})(\sqrt{x^2})$
- $(\sqrt{x})(\sqrt{x+5})$
- $(\sqrt{3x})(\sqrt{5x-4})$
- $(\sqrt{a+b})(\sqrt{a+b})$
- $(\sqrt{x-4})(\sqrt{x+5})$
- $(\sqrt{10})(\sqrt{5})$
- $(\sqrt{12x^3})(\sqrt{15x^5y})$
- $(\sqrt{xyz^4})(\sqrt{x^3y^5z})$
- $(\sqrt{28a^3})(\sqrt{10ab^6})$
- $(\sqrt{ab})(\sqrt{ac})$
- $(\sqrt{8a^2b})(\sqrt{4ab^2})$
- $(\sqrt{15xy^2})(\sqrt{10x^2y})$
- $(\sqrt{27m^2n^3})(\sqrt{18mn^4})$
- $(\sqrt{12a^3b^2})(\sqrt{8ab})$
- $(\sqrt{40ab})(\sqrt{10a^2b^4})$
- $(\sqrt{7a})(\sqrt{28ab})$
- $(\sqrt{12xy})(\sqrt{48x})$
- $(\sqrt{8x^2y})(\sqrt{50x^3y^2})$
- $(\sqrt{54a^2b^6})(\sqrt{9ab^2})$
- $(\sqrt{14a^7b^3})(\sqrt{6a^5b})$
- $(\sqrt{15ab^9})(\sqrt{5a^3b^5})$
- $(\sqrt{8ab^5c^7})(\sqrt{22a^8b^5c})$
- $(\sqrt{6ab})(\sqrt{30a})$
- $(\sqrt{24xy^3z^5})(\sqrt{30x^5y^3z^8})$
- $(\sqrt{18x^{12}y^5z^4})(\sqrt{15x^7y^2z^3})$
- $(\sqrt{15x^3})(\sqrt{12x^2})(\sqrt{3x^5})$
- $(\sqrt{5y^2})(\sqrt{30y^3})(\sqrt{10y^5})$
- $(\sqrt{6c^4})(\sqrt{3c^5})(\sqrt{4c^3})$
- $[\sqrt{4(x-1)}][\sqrt{5(x-1)^2}]$
- $[\sqrt{(x-2)^3}][\sqrt{(x-2)^2}]$
- $[\sqrt{6(x-4)^8}][\sqrt{7(x-4)}]$
- $[\sqrt{3(x-8)^5}][\sqrt{6(x-8)^7}]$
- $[\sqrt{8(x-7)^4}][\sqrt{9(x-7)^3}]$
- $[\sqrt{(x-3)^2}][\sqrt{4(x-3)^8}]$
- $[\sqrt{7(x-7)^9}][\sqrt{8(x-7)^6}]$
- $[\sqrt{2(x-9)^5}][\sqrt{8(x-9)^3}]$
- $[\sqrt{5(x-5)^6}][\sqrt{9(x-5)^2}]$

DIVIDING RADICALS

Just like in regular division, radicals must be simplified to complete the answer.

Radicals of the same root may be divided under the same radical or separated under different radicals.

Example: $\frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$

Example: $\frac{\sqrt{98}}{\sqrt{8}} = \sqrt{\frac{98}{8}}$ reduce fraction by 2 $\sqrt{\frac{49}{4}} = \frac{7}{2}$

A **rational expression** (fraction) with a radical in the denominator has not been completely simplified. To simplify it, multiply the fraction by **one**, using a ratio made up only by the number or expression of the denominator.

Example: $\frac{\sqrt{3}}{\sqrt{x}} = \frac{\sqrt{3}}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{3x}}{x}$

Notice the original was multiplied by **one** using the term of the denominator: $\frac{\sqrt{x}}{\sqrt{x}}$. Notice also that, because it is an operation of “simplification”, the *value* of the rational number has not changed. In the example immediately below, the value of the starting fraction, $\frac{\sqrt{54}}{\sqrt{12}} = 2.121\dots$, is also the value of the ending fraction $\frac{3\sqrt{2}}{2} = 2.121\dots$, which has been simplified.

Example: $\frac{\sqrt{54}}{\sqrt{12}} = \sqrt{\frac{54}{12}}$ reduce fraction $\sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

Example: Simplify $\frac{\sqrt{5a}}{\sqrt{7b^3}} = \frac{\sqrt{5a}}{\sqrt{7b^3}} \times \frac{\sqrt{7b^3}}{\sqrt{7b^3}} = \frac{\sqrt{35ab^3}}{7b^3} = \frac{b\sqrt{35ab}}{7b^2} = \frac{\sqrt{35ab}}{7b^2}$

Practice:

Divide and simplify.

1. $\frac{\sqrt{50}}{\sqrt{2}}$

6. $\frac{\sqrt{12x^3}}{\sqrt{5y^2}}$

11. $\sqrt{\frac{8}{3x}}$

16. $\frac{\sqrt{48}}{\sqrt{3}}$

21. $\frac{\sqrt{66x^2y}}{\sqrt{18xy^3}}$

26. $\frac{\sqrt{6c}}{\sqrt{y}}$

2. $\frac{\sqrt{2}}{\sqrt{y}}$

7. $\frac{\sqrt{75}}{\sqrt{12}}$

12. $\sqrt{\frac{18a^7}{6b^4}}$

17. $\frac{\sqrt{45xb^2}}{\sqrt{2cd}}$

22. $\frac{\sqrt{5}}{\sqrt{19a}}$

27. $\frac{\sqrt{300}}{\sqrt{24y}}$

3. $\frac{\sqrt{8x^3}}{\sqrt{6y^4}}$

8. $\sqrt{\frac{14}{2a}}$

13. $\frac{\sqrt{3}}{\sqrt{2}}$

18. $\sqrt{\frac{a}{8b^7}}$

23. $\frac{\sqrt{18}}{\sqrt{32}}$

28. $\sqrt{\frac{56}{a^2}}$

4. $\frac{\sqrt{15x^4}}{\sqrt{2x}}$

9. $\sqrt{\frac{a}{b^2}}$

14. $\sqrt{\frac{7}{a}}$

19. $\frac{\sqrt{5}}{\sqrt{24}}$

24. $\frac{\sqrt{x^2}}{\sqrt{18}}$

29. $\frac{\sqrt{y}}{\sqrt{xz}}$

5. $\frac{5}{\sqrt{3x}}$

10. $\frac{30y}{\sqrt{50}}$

15. $\sqrt{\frac{250}{75}}$

20. $\frac{\sqrt{44}}{\sqrt{56}}$

25. $\sqrt{\frac{a}{b}}$

30. $\frac{\sqrt{330}}{\sqrt{6ab}}$

ADDITION AND SUBTRACTION OF RADICALS

In addition and subtraction, radicals are treated like bases or variables: We only add or subtract the radicals with the same value and root (radicand).

Example: Add $3\sqrt{2} + 5\sqrt{2}$

$$\begin{array}{c} 3 + 5 = 8 \\ \swarrow \quad \downarrow \quad \searrow \\ 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2} \end{array}$$

the radical does not change, add both

Example: Add or subtract $\sqrt{3} + 4\sqrt{3} - 2\sqrt{7}$

$$\begin{array}{c} 1 + 4 = 5 \\ \swarrow \quad \downarrow \quad \searrow \\ \sqrt{3} + 4\sqrt{3} - 2\sqrt{7} = 5\sqrt{3} - 2\sqrt{7} \end{array}$$

radical 7 is different, do not subtract

Example: Add or subtract $6\sqrt{5} - 10\sqrt{5} + \sqrt{5}$

$$\begin{array}{c} 6 - 10 + 1 = -3 \\ \swarrow \quad \downarrow \quad \searrow \\ 6\sqrt{5} - 10\sqrt{5} + \sqrt{5} = -3\sqrt{5} \end{array}$$

all radicals are the same, compute all

Practice:

Add or subtract.

- | | | | |
|--------------------------------|---|--|--|
| 1. $5\sqrt{3} + 12\sqrt{3}$ | 10. $18\sqrt{x} - 7\sqrt{x}$ | 19. $32\sqrt{72} - 5\sqrt{98}$ | 27. $3\sqrt{147} - 6\sqrt{12}$ |
| 2. $10\sqrt{5} - 9\sqrt{5}$ | 11. $\sqrt{y} + \sqrt{y} + 2\sqrt{y}$ | 20. $4\sqrt{28} + 4\sqrt{7}$ | 28. $7\sqrt{52} + 4\sqrt{52}$ |
| 3. $\sqrt{2} + 3\sqrt{2}$ | 12. $34\sqrt{50} - 8\sqrt{72}$ | 21. $7\sqrt{50} + \sqrt{200}$ | 29. $18\sqrt{135} - 11\sqrt{60}$ |
| 4. $7\sqrt{y} + 2\sqrt{y}$ | 13. $8\sqrt{32} - 7\sqrt{98}$ | 22. $9\sqrt{126x} - 7\sqrt{56x}$ | 30. $\sqrt{\frac{1}{6}} + \sqrt{\frac{2}{3}} - \sqrt{6}$ |
| 5. $4\sqrt{2} - \sqrt{8}$ | 14. $2\sqrt{x} + 4\sqrt{x} - 5\sqrt{x}$ | 23. $5\sqrt{2} + 5\sqrt{8}$ | 31. $\sqrt{4u} - \sqrt{u} - 5\sqrt{u}$ |
| 6. $8\sqrt{18} - 5\sqrt{2}$ | 15. $23\sqrt{6} - 13\sqrt{24}$ | 24. $5\sqrt{5} - 9\sqrt{45}$ | 32. $\sqrt{9r} - 8\sqrt{r} + \sqrt{4r}$ |
| 7. $12\sqrt{12} + \sqrt{27}$ | 16. $5\sqrt{48} + 6\sqrt{75}$ | 25. $\sqrt{2} + \sqrt{\frac{1}{2}} - \sqrt{8}$ | 33. $8\sqrt{44} - 7\sqrt{99}$ |
| 8. $15\sqrt{20} + 13\sqrt{45}$ | 17. $9\sqrt{45} - 27\sqrt{5}$ | 26. $9\sqrt{48} - \sqrt{27}$ | 34. $\sqrt{60} + 3\sqrt{15}$ |
| 9. $9\sqrt{7} + 6\sqrt{49}$ | 18. $14\sqrt{8x} - 12\sqrt{2x}$ | | |

Add or subtract.

- | | |
|--|---|
| 35. $\sqrt{9x-45} + \sqrt{16x-80} - \sqrt{x-5} - \sqrt{10x-50}$ | 39. $4x\sqrt{x^2y} - x\sqrt{x^2y^3} + 5\sqrt{y^3} - 7y\sqrt{y}$ |
| 36. $x\sqrt{5x-10} - 2x\sqrt{9x-18} + 5x\sqrt{12x-24}$ | 40. $10x\sqrt{20} - x\sqrt{45} + 3x\sqrt{80} - 2x\sqrt{125} + \sqrt{180}$ |
| 37. $3y\sqrt{x^3y} - y\sqrt{xy^3} + x\sqrt{x^3y} - 7xy\sqrt{xy}$ | 41. $c^2\sqrt{a^2c} + c^2\sqrt{a^2c} + c^2\sqrt{a^2c} + c^2\sqrt{a^2c}$ |
| 38. $\sqrt{a^5 - a^2} - \sqrt{9a^3 - 9} + 4\sqrt{16a - 16}$ | 42. $11\sqrt{12x-8} - 8\sqrt{27x-18}$ |