Section 8.1
Simplifying Radical Expressions

A radical expression is one that contains roots. The number under the radical sign is called the radicand.

$$\sqrt{81}$$

All positive, real numbers have roots, but negative numbers do not.
The *perfect squares* such as 4, 9, 16, 25, 36... all have roots that are whole numbers:

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

and all other positive numbers in between such as 2, 3, 5, 6, 7, 8, 10, 11... also have roots, but they are irrational numbers:

$\sqrt{2} = 1.4142...$

$\sqrt{3} = 1.7320...$

$\sqrt{5} = 2.2360...$

$\sqrt{x^2} = x$

$\sqrt{y^4} = y^2$

Negative numbers do not have roots. A **negative** root, multiplied by its identity, another **negative** root, produces a **positive** number ($-2 \times -2 = +4$). Thus, the square root of positive numbers can be both, positive and negative, leaving negative numbers without square roots.

Example: If $3 \times 3 = 9$ and $-3 \times -3 = 9$

then the square root of 9 can be both +3 and –3 $\sqrt{9} = \pm 3$

and the square root of $\sqrt{-9}$, for example, cannot be found.

Practice:
Simplify.

1. $\sqrt{49}$
2. $\sqrt{81}$
3. $\sqrt{144}$
4. $\sqrt{289}$
5. $\sqrt{324}$
6. $\sqrt{121}$
7. $\sqrt{-22}$
8. $\sqrt{39}$
9. $\sqrt{x^4}$
10. $\sqrt{120}$
11. $\sqrt{250}$
12. $\sqrt{18}$
13. $\sqrt{800}$
14. $\sqrt{340}$
15. $\sqrt{-90}$
16. $\sqrt{y^6}$
17. $\sqrt{500}$
18. $\sqrt{900}$
19. $\sqrt{9y^8}$
20. $\sqrt{16x^{10}}$

RADICAL EXPRESSIONS

A radical expression, also called a radicand, is any expression found under a radical ($\sqrt{x}$). Moreover, negative outcomes of radical expressions are not real numbers.

Example: What values of x will make $\sqrt{x-5}$ a real number?

When $x = 0, 1, 2, 3$ or 4, the radical is not a real number because the result is a negative radicand. However, x values of 5 or above are possible.

When $x = 0$ the radical is $\sqrt{-5}$ no answer because the radicand is negative
When $x = 1$ the radical is $\sqrt{-4}$ no answer because the radicand is negative
...
When $x = 4$ the radical is $\sqrt{-1}$ no answer because the radicand is negative
When $x = 5$ the radical is $\sqrt{0} = 0$ (first real number)

Answer: $x \geq 5$

Example: What values will make $\sqrt{x^2+2}$ a real number?

Because squaring a negative number gives always a positive answer, in the example above all real numbers (including all negative numbers) are values that would make $\sqrt{x^2+2}$ a real number.

Example: What values will make $\sqrt{x^2}$ a real number?

$$\sqrt{x^2} = |x| \quad (x \text{ could be any real number})$$

Example: What values will make $\sqrt{5x+7}$ a real number?

$$5x + 7 \geq 0$$
$$5x \geq -7$$
$$x \geq -\frac{7}{5}$$

Answer: $x \geq -\frac{7}{5}$

Example: What values will make $\sqrt{(x-3)^2}$ a real number?

$$\sqrt{(x-3)^2} = |x-3| \quad (x \text{ could be any real number})$$

Example: What values will make $\sqrt{\frac{1}{9}y^2}$ a real number?

$$\sqrt{\frac{1}{9}y^2} = \frac{1}{3}|y| \qquad (y \text{ could be any real number})$$

Example: What values will make $\sqrt{4x^2 - 12xy + 9y^2}$ a real number?

Factor first: $\sqrt{4x^2 - 12xy + 9y^2} = \sqrt{(2x - 3y)(2x - 3y)}$

$$\sqrt{(2x - 3y)(2x - 3y)} = |2x - 3y| \qquad (x \text{ and } y \text{ could be any real number})$$

Practice:
Find the value of the variable that would yield a real number for the expression.

1. $\sqrt{x - 8}$
2. $\sqrt{x + 5}$
3. $\sqrt{2x - 9}$
4. $\sqrt{7x}$
5. $\sqrt{x - 18}$
6. $\sqrt{x^2 + 12}$
7. $\sqrt{3x - 4}$
8. $\sqrt{x^2 + 6}$
9. $\sqrt{x - 1}$
10. $\sqrt{x + 20}$
11. $\sqrt{y^2 - 4}$
12. $\sqrt{5z - 12}$

Simplify.

13. $\sqrt{y^4}$
14. $\sqrt{(x + 8)^2}$
15. $\sqrt{\frac{16}{25}y^2}$
16. $\sqrt{(x - y)^2}$
17. $\sqrt{x^2 - 12xy + 36y^2}$
18. $\sqrt{16x^2 - 40xy + 25}$
19. $\sqrt{(-a)^2}$
20. $\sqrt{\frac{81}{4}y^2}$
21. $\sqrt{x^2 + 2xy + y^2}$
22. $\sqrt{(-16b)^2}$
23. $\sqrt{(x + 4)^2}$
24. $\sqrt{\frac{(y^2 - 9)^2}{9}}$
25. $\sqrt{25x^2 + 90xy + 81}$
26. $\sqrt{(-49b)^2}$
27. $\sqrt{9(a - 12)^4}$

SIMPLIFICATION OF RADICALS (for nonnegative real numbers)
To simplify a radical means to reduce the radical to the point of not having any perfect squares represented in the radicand. In other words, every two of the same number—or base—under the radical sign, represents one outside the radical sign, the rest stays under the radical.

Example: Simplify $\sqrt{18}$

Use prime factorization to simplify radical $\sqrt{18} = \sqrt{2 \times 3 \times 3}$ (simplify 3^2)

Answer: $3\sqrt{2}$

Example: Simplify $\sqrt{16b^2}$

Use prime factorization to simplify radical $\sqrt{16b^2} = \sqrt{4 \times 4 \times b \times b} = 4b$

Perfect square, radical sign gone

Example: Simplify $\sqrt{25b}$

Use prime factorization to simplify radical $\sqrt{25b} = \sqrt{5 \times 5 \times b} = 5\sqrt{b}$

Example: Simplify $\sqrt{125a^3}$

Use prime factorization to simplify radical $\sqrt{125a^3} = \sqrt{5 \times 5 \times 5 \times a \times a \times a} = 5a\sqrt{5a}$

Example: Simplify $\sqrt{y^7}$

Use prime factorization to simplify radical $\sqrt{y^7} = \sqrt{yyyyyyy} = y^3\sqrt{y}$

Example: Simplify $\sqrt{3a^2 + 30a + 75}$

Factor the 3 first $\sqrt{3(a^2 + 10a + 25)}$

Simplify the perfect trinomial square $\sqrt{3(a+5)(a+5)} = (a+5)\sqrt{3}$

Practice:
Simplify. Assume all variables to be nonnegative.

1. $\sqrt{12}$
2. $\sqrt{27x^2}$
3. $\sqrt{32z}$
4. $\sqrt{98a^5}$
5. $\sqrt{a^4b^3c}$
6. $\sqrt{18x^2 - 60x + 50}$
7. $\sqrt{75}$
8. $\sqrt{60y^4}$
9. $\sqrt{250x^2y^3z^4}$
10. $\sqrt{(a+b)^2}$
11. $\sqrt{m^4n^7}$
12. $\sqrt{4a^2 + 56a + 196}$
13. $\sqrt{200}$
14. $\sqrt{48g^5h^3}$
15. $\sqrt{72b^5c^3}$
16. $\sqrt{36(x+y)^3}$
17. $\sqrt{45a^3b^2c^8}$
18. $\sqrt{20x^2 + 60x + 45}$
19. $\sqrt{80}$
20. $\sqrt{64wy^5}$
21. $\sqrt{xy^9}$
22. $\sqrt{(x-3)^3}$
23. $\sqrt{(2x+5)^7}$
24. $\sqrt{(a+8)^4}$
25. $\sqrt{4x^2 - 12x + 9}$
26. $\sqrt{(a+b)^5}$
27. $\sqrt{(6x-17)^3}$
28. $\sqrt{(x-8)^2}$
29. $\sqrt{9x^2 - 30xy + 25}$
30. $\sqrt{(s+t-u)^6}$
31. $\sqrt{(x+1)^{13}}$
32. $\sqrt{(8x+3)^7}$
33. $\sqrt{(2a+9)^3}$
34. $\sqrt{x^2 + 4x + 4}$
35. $\sqrt{x^5y^8}$
36. $\sqrt{(5x-12)^4}$
37. $\sqrt{(x+2)^8}$
38. $\sqrt{9x^2 + 6xy + 1}$
39. $\sqrt{\frac{(y^2-4)^3}{25}}$
40. $\sqrt{4x^2 - 8x + 4}$
41. $\sqrt{(4-b)^7}$