Section 7.5
Factoring Trinomials: ($ax^2 + bx + c$)

The way to factor a trinomial with a leading coefficient greater than one is similar to factoring a trinomial with a leading coefficient of one, except that now you also have to consider the factors of the product of the leading (first term) coefficient.

Example: Factor $2x^2 + 9x + 10$

Because the last term is positive, the two binomials will be additions.

Determine: Which two integers give a product of 10 and a sum of 9, considering that the first factors must give a product of 2?

Possible first factors: 2×1

Possible last factors: 10×1 5×2

Possible combinations:
$(2x + 10)(x + 1)$
$(2x + 1)(x + 10)$
$(2x + 5)(x + 2)$ Correct!
$(2x + 2)(x + 5)$

When using F.O.I.L., all combinations yield the first and last term of the trinomial correctly, but only the third one the correct middle term.

$2x^2 + 4x + 5x + 10$ $2x^2 + 9x + 10$

Answer: $(2x + 5)(x + 2)$

Example: Factor $4x^2 - 4x - 15$

Because the last term is negative, one binomial will be addition, the other one will be subtraction.

Determine: Which two integers give a product of –15 and a difference of –4, considering that the first factors must give a product of 4?

Possible first factors: 2×2 4×1

Possible last factors: -15×1 -5×3

Possible combinations:
1. $(2x - 15)(2x + 1)$
2. $(2x - 5)(2x + 3)$ Correct!
3. $(4x - 15)(x + 1)$
4. $(4x + 1)(x - 15)$
5. $(4x + 5)(x - 3)$
6. $(4x + 3)(x - 5)$

When using F.O.I.L., all combinations yield the first and last term of the trinomial correctly, but only the second one has the correct middle term.

Answer: $(2x - 5)(2x + 3)$

Example: Factor $6x^2 + x - 12$

Because the last term is negative, one binomial will be addition, the other one will be subtraction.

Determine: Which two integers give a product of –12 and a difference of 1, considering that the first factors must give a product of 6?

Possible first factors: 6×1 and 2×3

Last possible factors: 12×1 6×2 4×3

Possible combinations:

1. $(6x + 12)(x - 1)$
2. $(6x - 1)(x + 12)$
3. $(6x - 6)(x + 2)$
4. $(6x - 2)(x + 6)$
5. $(6x - 4)(x + 3)$
6. $(6x - 3)(x + 4)$
7. $(2x + 12)(3x - 1)$
8. $(2x - 1)(3x + 12)$
9. $(2x + 6)(3x - 2)$
10. $(2x - 2)(3x + 6)$
11. $(2x + 4)(3x - 3)$
12. $(2x + 3)(3x - 4)$ Correct!

When using F.O.I.L., all combinations yield the first and last term of the trinomial correctly, but only the twelfth one produces the correct middle term ($-8x + 9x = x$).

Answer: $(2x + 3)(3x - 4)$

Practice:
Factor completely (look for a common factor first.)

1. $2x^2 + 9x + 4$
2. $2x^2 + 7x + 5$
3. $3x^2 + 10x + 3$
4. $3x^2 + 10x + 7$
5. $3x^2 + 13x + 12$
6. $5x^2 + 17x + 6$
7. $5x^2 + 12x - 9$
8. $2x^2 + 11x + 14$
9. $3x^2 + 5x - 12$
10. $3x^2 + x + 10$
11. $2x^2 + 9x - 5$
12. $4a^2 + 15a + 14$
13. $2x^2 + 7x - 15$
14. $2x^2 + 11x + 12$
15. $3a^2 + 10a + 8$
16. $5x^2 + 3x - 14$
17. $6x^2 + 23x + 20$
18. $7p^2 + 25p - 12$
19. $7c^2 + 8c + 1$
20. $2x^2 + 15x - 8$
21. $3r^2 + 24r - 27$
22. $5x^2 + 6x - 8$
23. $4x^2 + 10x + 6$
24. $5x^2 - x - 22$
25. $2x^2 + 6x - 8$
26. $6y^2 - 11y + 3$
27. $8x^2 - 22x + 15$
28. $7y^2 - 15y - 18$
29. $10x^2 + 12x - 16$
30. $3m^2 - 15m + 12$
31. $3x^2 - 16x + 21$
32. $5n^2 - n - 22$
33. $7d^2 - 2d - 32$
34. $28x^2 - 25x - 42$
35. $25z^2 - 9z - 16$
36. $24x^2 - 6x - 18$
37. $32u^2 - 20u - 12$
38. $3j^2 - 36j + 81$
39. $7k^2 + 21k - 28$
40. $8x^2 + 16x - 24$
41. $10x^2 - 20x - 30$
42. $12y^2 - 20y - 48$
43. $32x^2 + 56x - 36$
44. $24g^2 - 9g - 15$
45. $14x^2 + 68x - 10$
46. $10h^2 - 54h - 36$
47. $12x^2 + 48xy - 144y^2$
48. $8x^2 - 48xy + 22y^2$
49. $7a^2 - 35ab + 28b^2$
50. $10x^2 + 25xy - 35y^2$
51. $15c^2 - 9cd - 24d^2$