Section 7.4
Factoring Trinomials: ($x^2 + bx + c$)

A trinomial with a leading coefficient of one is factored by creating two binomials in parentheses and determining the correct addition or subtraction (to match the middle term) of the products of the two inside and two outside values that form the two binomials.

Sum of 7 ↘ ↙ Product of 12

Example: Factor the trinomial $x^2 + 7x + 12$

Determine: Which two integers (when multiplied) give a product of 12 and (when added) a sum of 7?

There are only three possible multiplications that yield a 12: 4×3 6×2 12×1

Of these three, the correct sum is the one that adds up (3 + 4) to 7. Answer: $(x + 3)(x + 4)$

Using F.O.I.L. to check the answer:
—First: $(x)(x) = x^2$
—Outside: $(x)(4) = 4x$
—Inside: $(3)(x) = 3x$
—Last: $(3)(4) = 12$

Add like terms
$3x + 4x = 7x$

$x^2 + 7x + 12$ (check!)

Note: When the last term is positive and the middle term is negative, then both binomials are subtractions.

Example:
Factor the above example with a negative middle term.

$x^2 - 7x + 12$ Answer: $(x - 3)(x - 4)$

Using F.O.I.L. to check the answer:
—First: $(x)(x) = x^2$
—Outside: $(x)(-4) = -4x$
—Inside: $(-3)(x) = -3x$
—Last: $(-3)(-4) = 12$

Add like terms
$-3x + -4x = -7x$

$x^2 - 7x + 12$ (check!)

When to add, when to subtract
Whether we add or subtract to form the two binomials is **determined by the sign of the last term**. If the sign of the last term is positive, the factors of the product of the last term are added. If the sign of the last term is negative, the factors of the product of the last term are subtracted.

Example: Factor $y^2 + y - 20$

Determine: Which two integers (if multiplied) give a product of –20 and (if subtracted) a difference of 1?

Possible factors: -1×20 -2×10 -4×5

only the difference of $5 - 4 = 1$ fits Answer: $(y - 4)(y + 5)$

Example: Factor $y^2 - 5y - 6$

Determine: Which two integers give a product of –6 and a difference of –5?

Possible factors: -6×1 -2×3

only the difference of $1 - 6 = -5$ fits Answer: $(y - 6)(y + 1)$

Example: Factor $y^2 - 11y + 18$

Determine: Which two integers give a product of 18 and a sum of –11?

Possible factors: $(-18)(-1)$ $(-2)(-9)$ $(-3)(-6)$

only the sum of $-2 - 9 = -11$ fits Answer: $(y - 9)(y - 2)$

Practice:
Factor completely (look for a common factor first.)

1. $x^2 + 3x + 2$
2. $x^2 + 4x + 3$
3. $x^2 + 6x + 5$
4. $x^2 + 5x + 4$
5. $x^2 + 7x + 10$
6. $x^2 + 5x + 6$
7. $x^2 + 4x - 45$
8. $x^2 + 7x + 12$
9. $x^2 + x - 12$
10. $x^2 + 11x + 10$
11. $x^2 + 4x - 5$
12. $a^2 + 9a + 18$
13. $x^2 + 2x - 15$
14. $x^2 + 8x + 12$
15. $a^2 + 6a + 8$
16. $x^2 + 5x - 14$
17. $x^2 + 9x + 20$
18. $p^2 + 5p - 24$
19. $c^2 + 10c + 21$
20. $x^2 + 3x - 28$
21. $r^2 + 12r + 27$
22. $x^2 + 2x - 8$
23. $t^2 + 10t + 16$
24. $x^2 - 3x + 2$
25. $x^2 + 7x - 8$
26. $y^2 - 4y + 3$
27. $x^2 - 6x + 5$
28. $y^2 - 6y - 7$
29. $x^2 + x - 6$
30. $m^2 - 5m + 14$
31. $x^2 - 10x + 21$
32. $n^2 - 9n - 22$
33. $d^2 - 12d + 32$
34. $x^2 - x - 42$
35. $z^2 - 10z + 16$
36. $x^2 - 7x - 18$
37. $u^2 - 8u - 9$
38. $j^2 - 9j + 8$
39. $k^2 + 4k - 12$
40. $x^2 + 10x + 24$
41. $x^2 - x - 30$
42. $y^2 - 12y - 13$
43. $x^2 - 13x + 36$
44. $g^2 - 2g - 15$
45. $x^2 + 3x - 10$
46. $h^2 - 9h - 36$
47. $x^2 + 11xy - 12y^2$
48. $x^2 - 13xy + 22y^2$
49. $a^2 - 14ab + 24b^2$
50. $x^2 + 4xy - 32y^2$
51. $c^2 - 7cd - 8d^2$
52. $x^2 + 9xy - 10y^2$
53. $x^2 - 3xy - 18y^2$
54. $m^2 + mn - 20n^2$
55. $x^2 - 3xy - 28y^2$
56. $w^2 - 16wz - 36z^2$
57. $x^2 - 26xy + 25y^2$
58. $x^2 + 15xy - 34y^2$
59. $a^2 - 6ab - 40b^2$
60. $x^2 - 7xy - 44y^2$
61. $x^2 + 13xy - 30y^2$
62. $c^2 - cd - 56d^2$
63. $x^2 + 14xy - 15y^2$
64. $c^2 - 12cd + 20d^2$
65. $x^2 - 6xy - 55y^2$
66. $a^2 - 8ab - 48b^2$
67. $x^2 + 12xy - 28y^2$
68. $w^2 - 5wz - 50z^2$
69. $x^2 + 3xy - 70y^2$