Section 7.3
Perfect Trinomial Squares

Like the *difference of two squares*, a *perfect trinomial square* is formed by two squares, but unlike the *difference of two squares*, it's a trinomial and not a binomial.

HOW TO RECOGNIZE A TRINOMIAL THAT IS A PERFECT SQUARE

—The last term must be positive
—The first and third term (corners) must be squares
—The middle term is twice the product of the square roots of the first and last terms (corners).

Examples: $x^2 + 2x + 1$ is a *perfect trinomial square* because the last term is positive

the first and last terms are squares ($\sqrt{x^2} = x$ and $\sqrt{1} = 1$)

the middle term is $2(x \times 1) = 2x$

twice

$4x^2 + 12x + 9$ is a *perfect trinomial square* because the last term is positive

the first and last terms are squares ($\sqrt{4x^2} = 2x$ and $\sqrt{9} = 3$)

the middle term is $2(2x \times 3) = 12x$

twice

$25y^2 - 45y - 36$ is NOT a *perfect trinomial square* because the last term is negative.

$16a^2 + 30a + 9$ is NOT a *perfect trinomial square* because the middle term is not twice the product of the square roots of the first and last terms $2(4a \times 3) = 24a$

FACTORING A PERFECT TRINOMIAL SQUARE

To factor a *perfect trinomial square*, set up two binomials in parentheses. If the middle term is addition, both binomials are additions, if the middle term is subtraction, both binomial are subtractions. Find the square root of the first term and third term. Place the square roots of the first square to start each parenthesis, and the square roots of the second square to end each parenthesis. Because the middle term is addition, the first example above, $x^2 + 2x + 1$ is factored into

$$(x + 1)(x + 1)$$

Because both binomials are identical, square them to: $(x + 1)^2$

The second example above $4x^2 + 12x + 9$ can be factored to $(2x + 3)(2x + 3) = (2x + 3)^2$

Example: Factor $25x^2 - 60xy + 36y^2$

First term: $\sqrt{25x^2} = 5x$ second term: $\sqrt{36y^2} = 6y$ middle term: $(2)(5x \times 6y) = 60xy$

It is a perfect trinomial square. Because the middle term of the original trinomial is subtraction, both binomials will be subtractions.

Answer: $(5x - 6y)(5x - 6y) = (5x - 6y)^2$

Practice:
Factor completely (look for a common factor first.)

1. $x^2 + 4x + 4$
2. $x^2 + 10x + 25$
3. $y^2 + 6y + 9$
4. $a^2 + 18a + 81$
5. $x^2 + 20x + 100$
6. $y^2 + 8y + 16$
7. $y^2 - 14y + 49$
8. $x^2 + 12x + 36$
9. $y^2 - 22y + 121$
10. $x^2 + 16x + 64$
11. $y^2 + 26y + 169$
12. $b^2 + 30b + 225$
13. $x^2 - 24x + 144$
14. $4x^2 + 4x + 1$
15. $9x^2 + 12x - 4$
16. $64y^2 + 80y + 25$
17. $25y^2 - 40y + 16$
18. $16a^2 + 24a + 9$
19. $4x^2 - 16x + 16$
20. $64y^2 + 48y + 9$
21. $16x^2 + 64x + 64$
22. $9b^2 + 30b + 25$
23. $25y^2 + 90y + 81$
24. $81y^2 + 18y + 1$
25. $36y^2 - 84y + 49$
26. $x^2 + 6xy + 9y^2$
27. $x^2 - 8xy + 16y^2$
28. $a^2 + 18ab + 81b^2$
29. $x^2 + 20xy + 100y^2$
30. $x^2 + 16xy + 64y^2$
31. $c^2 + 10cd + 25d^2$
32. $x^2 + 12xy + 36y^2$
33. $m^2 + 14mn + 49n^2$
34. $x^2 + 24xy + 144y^2$
35. $x^2 + 4xy - 4y^2$
36. $w^2 + 30wz + 225z^2$
37. $x^2 + 22xy + 121y^2$
38. $p^2 + 2pq + q^2$
39. $4x^2 + 12xy + 9y^2$
40. $25x^2 + 40xy + 16y^2$
41. $16a^2 + 72ab + 81b^2$
42. $64x^2 + 160xy + 100y^2$
43. $25x^2 - 80xy + 64y^2$
44. $9c^2 + 30cd + 25d^2$
45. $49x^2 + 84xy + 36y^2$
46. $4m^2 + 28mn + 49n^2$
47. $81x^2 + 90xy + 25y^2$
48. $25x^2 + 20xy - 4y^2$
49. $4w^2 - 60wz + 225z^2$
50. $9x^2 + 66xy + 121y^2$
51. $2x^2 + 8x + 8$
52. $2x^2 + 20x + 50$
53. $2y^2 - 12y + 18$
54. $3a^2 + 54a + 243$
55. $2x^2 + 40x + 200$
56. $3y^2 + 24y + 48$
57. $3y^2 + 42y + 147$
58. $2x^2 + 24x + 72$
59. $5y^2 + 5y + 5$
60. $5x^2 + 10x + 20$
61. $7y^2 + 42y + 63$
62. $2b^2 + 60b + 450$
63. $4x^2 - 32x + 64$
64. $32x^2 + 32x + 8$
65. $18x^2 + 24x - 8$
66. $640y^2 + 800y + 250$
67. $50y^2 + 80y + 32$
68. $48a^2 + 72a + 27$
69. $8x^2 - 32x + 32$
70. $0.16a^2 + 0.24a + 0.09$
71. $0.09b^2 + 0.3b + 0.25$
72. $0.25y^2 + 0.9y + 0.81$
73. $\frac{1}{4}y^2 + y + 1$
74. $\frac{1}{9}y^2 + \frac{1}{3}y + \frac{1}{4}$