Section 7.2
Difference of Two Squares

A *difference of two squares* is a binomial that can be factored into two other binomials. Because a square, by definition, is the product of two identical numbers (for instance, 16 is the square of 4 × 4, 9 is the square of 3 × 3), the *difference of two squares* is two squares with a subtraction sign between them.

Examples: $x^2 - 4$ is a *difference of squares* because both x^2 and 4 are squares.

$9y^2 - 25$ is a *difference of squares* because 9, y^2 and 25 are squares.

FACTORING A DIFFERENCE OF TWO SQUARES

To factor a *difference of two squares*, set up two binomials in parenthesis: One separated by addition, another one separated by subtraction. Place the square roots of the first square to start each parenthesis, and the square roots of the second square to end each parenthesis. The first example above, $x^2 - 4$, is factored into

$(x + 2)(x - 2)$ because the $\sqrt{x^2} = x$ and the $\sqrt{4} = 2$

The factors of the second example above, $9y^2 - 25$, are $(3y + 5)(3y - 5)$

because the $\sqrt{9} = 3$, the $\sqrt{y^2} = y$, and the $\sqrt{25} = 5$

Example: Factor $49a^2 - 100$

Answer: $(7a + 10)(7a - 10)$ because the $\sqrt{49} = 7$, the $\sqrt{a^2} = a$, and the $\sqrt{100} = 10$

Check work by multiplying the two factored binomials. Multiplication takes us back to the original binomial:

Multiply $(7a + 10)(7a - 10)$

using F.O.I.L. $(7a)(7a) + (7a)(-10) + (10)(7a) + (10)(-10)$

$49a^2 - 70a + 70a - 100$

cancelling $-70a + 70a$, we get back the original $49a^2 - 100$

FACTORING MORE THAN ONCE

Factoring **completely** means that in some occasions factoring is not finished on the first try and some more factoring should be done.

Example: Factor $x^4 - 16$

Because it is a difference of squares, you factor it into two binomials (one addition and one subtraction):

$$(x^2 + 4)(x^2 - 4)$$

The first binomial is a "sum of squares" (black) and cannot be factored. The second binomial (red) is another *difference of two squares*, thus turning the "complete factoring" into:

$$(x^2 + 4)(x^2 - 4)$$

$$\rightarrow (x^2 + 4)(x + 2)(x - 2)$$

Example: Factor $x^8 - 1$

First round	$(x^4 + 1)(x^4 - 1)$
Second round	$(x^4 + 1)(x^2 + 1)(x^2 - 1)$
Third round	$(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$

Practice:
Factor completely (look for a common factor first.)

1. $x^2 - 4$
2. $a^2 - b^2$
3. $x^2 - 25$
4. $a^2 - 9$
5. $9p^2 - 4$
6. $4x^2 - 25y^2$
7. $9x^2 - y^2$
8. $16x^2 + 9y^2$
9. $25x^2 - 49y^2$
10. $x^2 - y^2$
11. $x^2 - 16y^2$
12. $4x^2 - 9y^2$
13. $25x^2 - 100$
14. $9x^2 - 169$
15. $36x^2 - 25y^2$
16. $50x^2 - 98y^2$
17. $8x^2 - 50y^2$
18. $16x^2 - 81y^2$
19. $36x^2 - y^2$
20. $25x^2 - 121$
21. $49x^2 - 9y^2$
22. $98x^2 - 32y^2$
23. $27x^2 + 48y^2$
24. $5x^2 - 45y^2$
25. $x^2 - 100$
26. $x^2 - 225$
27. $4x^2 - 9y^2$
28. $81x^2 - 25y^2$
29. $36s^2 - 16t^2$
30. $18x^2 - 200y^2$
31. $18x^2 + 50y^2$
32. $72x^2 - 8y^2$
33. $12u^2 - 75v^2$
34. $7x^2 - 63y^2$
35. $25x^2 - 1$
36. $16x^4 - 1$
37. $49a^2 - b^2$
38. $1 - 25y^2$
39. $4a^2 - 144b^2$
40. $8x^2 - 50y^2$
41. $12x^2 - 27y^2$
42. $1 - 9y^2$
43. $18w^2 - 128z^2$
44. $1 - 36y^2$
45. $-25x^2 + 4$
46. $100x^2 - 1$
47. $72x^2 + 25y^2$
48. $25x^2 - 9y^2$
49. $-27x^2 + 48$
50. $-32x^2 - 200y^2$
51. $1 - 4y^2$
52. $a^2 - 121b^2$
53. $8x^2 - 98$
54. $-16x^4 + 1$
55. $18x^2 - 288y^2$
56. $9x^2 - 16y^2$
57. $36x^2 - 9y^2$
58. $75x^2 - 3$
59. $8c^2 - 2$
60. $-48x^4 + 3y^4$
61. $8x^2 + 225y^2$
62. $11x^2 - 99y^2$
63. $90c^2 - 250d^2$
64. $5x^2 - 125y^2$
65. $6x^2 - 24y^2$
66. $5x^2 - 20y^2$
67. $4x^2 - 225y^2$
68. $16m^2 - 169n^2$
69. $9x^2 + 100y^2$
70. $-x^2 + 81y^2$
71. $x^2 - 64$
72. $81x^2 - 16$