

Section 7.1

Common Factors in Polynomials

In mathematics, factors are the parts that make a whole, and factoring involves division because they are the numbers you get when you divide a number or expression (a given quantity) without a remainder.

The FACTORS of numbers are always smaller than the numbers.

Example: If we divide $\frac{6}{2} = 3$, the factors of **6** are **2** and **3**.

To find ALL the factors of a number (called *prime factorization*), divide the number by each prime number starting with the number 2, until you can't divide again without getting a remainder (decimal.)

Example: Find all the factors of 54

$$\frac{54}{2} = \frac{27}{3} = \frac{9}{3} = 3$$

Answer: The factors of 54 are $(2)(3)(3)(3) = (2)(3^3)$

Example: Factor $18 + 36 + 54$ completely

The prime factorization of 18 is $(2)(3)(3)$

The prime factorization of 36 is $(2)(2)(3)(3)$

The prime factorization of 54 is $(2)(3)(3)(3)$

2 is common to all three numbers once: 2

3 is common to all three numbers twice: $(3)(3) = 9$

$$(9)(2) = 18$$

Because the highest common factor for all three numbers is 18, the expression $18 + 36 + 54$ can be factored to:

$$18(1 + 2 + 3)$$

By dividing $\frac{18}{18}$, $\frac{36}{18}$, and $\frac{54}{18}$, 18 has been factored to show that $18 + 36 + 54$ can also be written as the product of

$$18(1 + 2 + 3)$$

In algebra, factoring works the same way, except this time with numbers AND symbols.

Example: Factor $18a^3 + 36a^2 + 54a$ completely

The factors of $18a^3$ are $(2)(3)(3)(a)(a)(a)$

The factors of $36a^2$ are $(2)(2)(3)(3)(a)(a)$

The factors of $54a$ are $(2)(3)(3)(3)(a)$

Common in all three terms we find $(2)(3)(3)(a) = 18a$

Because the highest common factor for all three numbers is $18a$, the expression $18a^3 + 36a^2 + 54a$ can be factored:

$$18a(a^2 + 2a + 3)$$

By dividing $\frac{18a^3}{18a} = a^2$, $\frac{36a^2}{18a} = 2a$, and $\frac{54a}{18a} = 3$, $18a$ has been factored to show that:

$$18a^3 + 36a^2 + 54a \quad \text{can also be written as the product of} \quad 18a(a^2 + 2a + 3)$$

Example: Factor completely $4x^6 + 6x^4 - 14x^3 + 2x^2$

The common coefficient found in 4, 6, 14, and 2 is **2**.

The common base found **x**.

The common exponent found is **2**.

The common factor is then $2x^2$.

By dividing each term of the expression by $2x^2$

$$4x^6 \text{ becomes } 2x^4$$

$$6x^4 \text{ becomes } 3x^2$$

$$14x^3 \text{ becomes } 7x$$

$$2x^2 \text{ becomes } 1$$

$$\text{Answer: } 2x^2(2x^4 + 3x^2 - 7x + 1)$$

Practice:

Factor completely.

1. $3x^3 + 6x^2 + 12x$
2. $5x^4 - 15x^2 + 25x$
3. $12x^3y^3 - 8x^2y^2 + 4xy$
4. $3x^2y^2 - 6x^3y^3 - 9x^2y^2$
5. $2x^6 + 4x^5 - 6x^4 + 8x^3$
6. $5y^5 - 10y^4 - 15y^3 + 20y^2$
7. $z^3 + z^4 + z^5 + z^7$
8. $4a^4 - 8a^3 - 12a^2 + 16a$
9. $3a^4b^4 + 6a^3b^3 - 9a^2b^2 - 12ab$
10. $10a^5b^5c^5 + 15a^3b^3c^3 - 20a^2b^2c^2 + 25abc$
11. $6x^4y^4z^4 - 12x^6y^4z^3 - 18x^6y^4z^3 + 24xyz$
12. $2a^2b^2 + 6a^4b^4 + 8a^5b^5 - 14a^3b^5$
13. $3x^6y^3z^6 + 5x^7y^3z^5 - 4x^3y^3z^3 + 8xyz$
14. $3c^5d^5 - 6c^7d^7 + 9c^6d^6 + 18c^8d^8$
15. $5x^8y^7z + 5x^5y^4z^3 - 15x^4y^3z^2 - 35$
16. $20e^5f + 18e^6f^8 - 10e^7f^4 + 36$
17. $5x^3y^3z^3 - 15x^5y^4z^3 + 10x^2y^9z^8 + 5x^2y^5z^7$
18. $21g^8h^5 + 7g^5h^6 - 14g^7h^8 + 28g^8h^9$
19. $21x^6y^6z^7 - 9x^3y^2z^5 + 6x^7y^7z^7 - 3x^2y^3z$
20. $5j^3k^3 + 10j^2k^8 - 5j^6k^7 + 17j^7k^5$
21. $6x^7y^7z^7 + 9x^8y^7z^6 - 9x^5y^4z^{11} + 18$
22. $20m^2n^4 + 16m^3n^5 - 12m^7n^8 + 4m^7n^5$
23. $40x^2y^3z^4 - 30x^5y^5z^5 + 20x^5y^7z^{10} - 10y^7z^8$
24. $21p^6q^2 + 24p^2q^4 - 14p^5q^9 + 7p^3q^8$
25. $24x^3y^3z^3 + 12x^9y^8z^7 + 18x^2y^3z^4 - 6$
26. $60s^6t^6 - 15s^4t^4 - 45s^5t^5 + 15s^3t^7$
27. $x^4y^5z^6 + x^7y^7z^7 - x^7y^8z^9 + x^6y^7z^8$
28. $14u^8v^{12} + 10u^9v^8 + 6u^7v^6 + 2u^5v^3$
29. $4x^{12}y^{12}z^{12} - 8x^8y^6z^2 - 4x^8y^6z^2 - 4x^8y^6z^2$
30. $18a^{14}b^{18} - 12a^{10}b^3 - 6a^3b^9 + 6a^7b^2$
31. $3x^4y^5z^6 + 4x^5y^{14}z^8 + 6x^7y^{15}z^8 + 8$
32. $2c^3d^3 - 5c^3d^3 - 6c^2d^2 - 8cd$
33. $6x^4y^8z^3 + 9x^7y^5z^3 + 12x^6y^7z^8 + 15x^3y^4z^5$
34. $17e^2f + 34e^3f^3 - 17e^2f^4 + 51e^2f$
35. $5x^3y^4z^9 + 5x^3y^8z^7 + 5xy^3z^4 - 5$
36. $7g^2h - 2g^9h^7 - 7g^3h^9 + 14g^7h^4$
37. $8xy^5z + 16x^2yz^3 - 14x^4y^5z^4 + 28x^2y^5z^8$
38. $9x^3y^5z - 36xy^3z^6 + 27x^2y^7z^6 - 18$