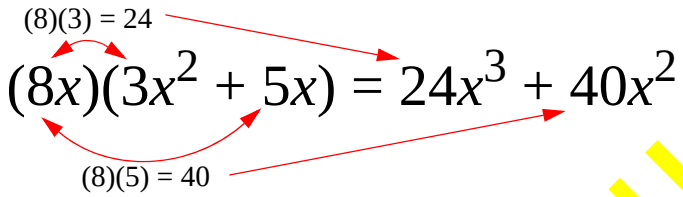
Section 6.4
Multiplication of Polynomials

MULTIPLYING A MONOMIAL BY A BINOMIAL

Similar to multiplication of integers, in the multiplication of polynomials we use the distributive property: Each term of the polynomial is multiplied by the monomial.

Example: Multiply $(8x)(3x^2 + 5x)$



Notice that the rules of multiplication of monomials have been applied: The exponents have been added (1 + 2 = 3) in the first product and (1 + 1 = 2) in the second one.

MULTIPLYING BINOMIALS

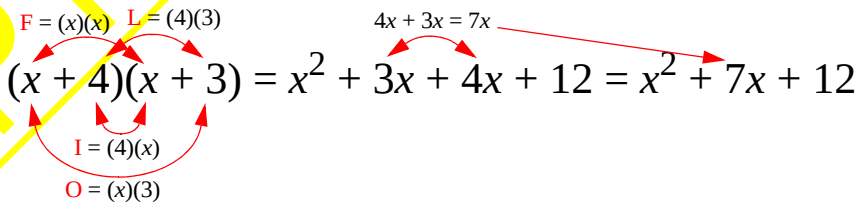
Multiplying binomials is often called the F.O.I.L. method, where F stands for *first* terms, O for *outside* terms, I for *inside* terms, and L for *last* terms.

Example: Multiply $(x + 4)(x + 3)$

See how the F.O.I.L. method is used:

- First: $(x)(x) = x^2$
- Outside: $(x)(3) = 3x$
- Inside: $(4)(x) = 4x$
- Last: $(4)(3) = 12$

Add like terms
$3x + 4x = 7x$



Example: Multiply $(3a + 5b)(2a - b)$

- First: $(3a)(2a) = 6a^2$
- Outside: $(3a)(-b) = -3ab$
- Inside: $(5b)(2a) = 10ab$

Add like terms
$-3ab + 10ab = 7ab$

- Last: $(5b)(-b) = -5b^2$

$$(3a + 5b)(2a - b) = 6a^2 - 3ab + 10ab - 5b^2 = 6a^2 + 7ab - 5b^2$$

SPECIAL PRODUCTS OF BINOMIALS

Special binomials are binomials that look similar. However, because the positive and negative signs are positioned differently, their products change considerably. They are:

1. $(a + b)(a + b)$
2. $(a - b)(a - b)$
3. $(a + b)(a - b)$

The product of the first one above (using FOIL) is:

$(a)(a) + (a)(b) + (a)(b) + (b)(b) = a^2 + ab + ab + b^2 = \mathbf{a^2 + 2ab + b^2}$

The product of the second one is:

$(a)(a) - (a)(b) - (a)(b) + (b)(b) = a^2 - ab - ab + b^2 = \mathbf{a^2 - 2ab + b^2}$

The product of the third one is:

$(a)(a) - (a)(b) + (a)(b) - (b)(b) = a^2 - ab + ab + b^2 = \mathbf{a^2 - b^2}$ — *difference of two squares*

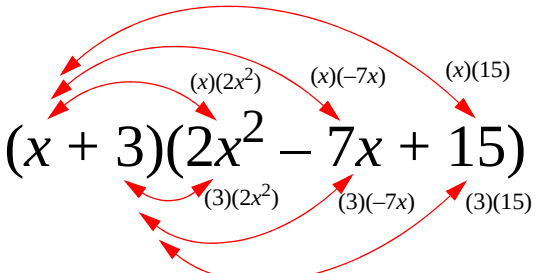
Notice that in the multiplication of the third one, $-ab$ and $+ab$ cancel out. Because of this, this particular "special product" is called *the difference of two squares*.

NOTE: The *sum of two squares*, $a^2 + b^2$, is not the product of any multiplication.

MULTIPLYING A BINOMIAL AND A TRINOMIAL

We continue to multiply each term in the first polynomial by every other term in the second polynomial.

Example: Multiply $(x + 3)(2x^2 - 7x + 15)$



First term of binomial times trinomial: $(x)(2x^2) + (x)(-7x) + (x)(15) = 2x^3 - 7x^2 + 15x$

Second term of binomial times trinomial: $(3)(2x^2) + (3)(-7x) + (3)(15) = 6x^2 - 21x + 45$

Combine the results of both multiplications: $2x^3 - 7x^2 + 15x + 6x^2 - 21x + 45$

Answer: $2x^3 - x^2 - 6x + 45$

MULTIPLYING TWO TRINOMIALS

We continue to multiply each term in the first polynomial by every other term in the second polynomial.

Example: Multiply $(2x^2 + 5x - 3)(3x^2 - 6x + 9)$

First term of first trinomial times second trinomial: $(2x^2)(3x^2) + (2x^2)(-6x) + (2x^2)(9) = 6x^4 - 12x^3 + 18x^2$

Second term of first trinomial times second trinomial: $(5x)(3x^2) + (5x)(-6x) + (5x)(9) = 15x^3 - 30x^2 + 45x$

Third term of first trinomial times second trinomial: $(-3)(3x^2) + (-3)(-6x) + (-3)(9) = -9x^2 + 18x - 27$

Combine all three products: $6x^4 - 12x^3 + 18x^2 + 15x^3 - 30x^2 + 45x - 9x^2 + 18x - 27$

Answer: $6x^4 + 3x^3 - 21x^2 + 63x - 27$

Practice:

Find the product.

1. $(x)(2x^2 + 4x + 5)$
2. $(3x)(x^3 + 7x^2 + 2x)$
3. $(9x^4 + 7x^3 + 3x^2)(x)$
4. $(8y^2 + 12y + 6)(2y^2)$
5. $(x + 2)(x + 3)$
6. $(x + 9)(x + 5)$
7. $(x + y)(x + y)$
8. $(y + 8)(2y + 4)$
9. $(3x + y)(2x - y)$
10. $(2x + 5)(x - 2)$
11. $(x - y)(x - y)$
12. $(x - 5y)(4x - 3y)$
13. $(x + y)(x - y)$
14. $(2a + 4b)(6a - 5b)$
15. $(3x - y)(x + 7y)$
16. $(5c - d)(9c + 8d)$
17. $(y - 6x)(3y - x)$
18. $(2x + 10y)(2x + 10y)$
19. $(s - 8t)(5s - 3t)$
20. $(2a - 12b)(2a - 12b)$
21. $(a + 3b)(5a - 8b)$
22. $(x - 8)(7x - 12)$
23. $(4x - 3y)(4x + 3y)$
24. $(a - 9b)(5a - 7b)$
25. $(7x - 8)(x - 15)$
26. $(5x - 6y)(5x + 6y)$
27. $(20x + 5)(5x + 20)$
28. $(3x - 4y)(9x - 8y)$
29. $(c + d)(c - d)$
30. $(4x + 12y)(10x - 7y)$
31. $(14x - 3y)(9x - y)$
32. $(3x - 11y)(x + 7y)$
33. $(6s - 5t)(2s + 12t)$
34. $(7a + 4b)(6a - 5b)$
35. $(12c - 6d)(3c - 11d)$
36. $(a - 7b)(2a - 14b)$
37. $(3p + 8q)(9p - 10q)$
38. $(7a + 5)(7a - 5)$
39. $(x - 18y)(18x - y)$
40. $(18a - 3b)(3a + 20b)$
41. $(3x^2 + 7)(2x^2 - 4)$
42. $(x + 7)(x^2 - 5x + 4)$
43. $(5x^2 - 8x - 2)(x + 9)$
44. $(x^2 - x + 6)(3x - 10)$
45. $(-4x^2 - 9x - 6)(x^2 - x)$
46. $(-2y^2 - y)(y^3 + 8y^2 + y)$
47. $(a^3 + a^2)(a^3 + a^2 + a)$
48. $(x^2 + x - 5)(x^2 + 7x - 8)$
49. $(x^2 + 4x - 10)(-x^2 + 9x + 3)$
50. $(y^2 + y - 4)(-y^2 - 6y + 2)$
51. $(3c^3 - 2c^2 + c)(c^3 + 5c^2 - c)$