

Section 6.3

Addition and Subtraction of Polynomials

In polynomials, algebraic addition and subtraction follow similar rules, and what is important in algebraic addition is the number line (see section 1.6). In the number line we compare negative and positive numbers and the answer is determined by the balance found.

Example: $8x + 9x - 3x - 7x + 2x = 9x$

To solve the above expression using the number line

- starting at zero
- move 8 to the right = 8
- move 9 to the right = 17
- move 3 to the left = 14
- move 7 to the left = 7
- move 2 to the right = 9

Example: $-12y + 5y + 2y - 6y + 16y - 7y = -2y$

Using the number line

- starting at zero
- move 12 to the left = -12
- move 5 to the right = -7
- move 2 to the right = -5
- move 6 to the left = -11
- move 16 to the right = 5
- move 7 to the left = -2

ADDITION OF POLYNOMIALS

Only the coefficients of monomials with the same base and the same exponent may be added or subtracted.

The expression

$$3a^2 + a$$

cannot be added. However, the expressions

$$3a^2 + a^2 = 4a^2 \quad \text{OR} \quad 3a + a = 4a \quad \text{may be added because they have the same base and exponent}$$

Example: Evaluate $3x + 5x - x + 6x = 13x$ Combine coefficients $3 + 5 - 1 + 6 = 13$

$$\begin{array}{ccccccc} & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 3x & + & 5x & - & x & + & 6x & = & 13x \end{array}$$

Example: Evaluate: $(3x^3 + 2x^2 - 5x) + (5x^3 + x^2 - 2x)$

Remove parentheses $3x^3 + 2x^2 - 5x + 5x^3 + x^2 - 2x$

Combine coefficients according to base and exponent

$$\begin{array}{l} 3x^3 + 5x^3 = 8x^3 \\ 2x^2 + x^2 = 3x^2 \\ -5x - 2x = -7x \end{array} \longrightarrow \text{Answer: } 8x^3 + 3x^2 - 7x$$

Example: $(-a^7 + 6a^4 - 4a^3) + (-5a^4 + 3a^3 - 8a^2)$

remove parentheses $-a^7 + 6a^4 - 4a^3 - 5a^4 + 3a^3 - 8a^2$

Combine coefficients according to base and exponent

$$\begin{array}{l} -a^7 \\ 6a^4 - 5a^4 = a^4 \\ -4a^3 + 3a^3 = -a^3 \\ -8a^2 \end{array} \longrightarrow \text{Answer: } -a^7 + a^4 - a^3 - 8a^2$$

Practice:

Add algebraically.

1. $(5x^2 + 3y^2 - z^2) + (2x^2 + 4y^2 - 2z^2)$
2. $(5x + 2y^3 - 3z^2) + (x - y^3 + z^2)$
3. $(a^3 + 6b^4 + 9c + 8) + (-7a^3 + 5b^4 + 3c)$
4. $(-4r^2 - s - 6t^4) + (4r^2 - s - 3t^4)$
5. $(2x - 9y^3 + 2z^5) + (8x + 3y^3 + 5z^5 + 7)$
6. $(-6c + 7b^2 - 5c^2 + 3) + (9c^2 + 7b^2 - 3c - 21)$
7. $(9p + 5q^3 - 8r^2) + (-6p + 16q^3 + 2r^2)$
8. $(x + y^2 + z^2 + 7) + (6x + 55y^2 + 3z^2 + 32)$
9. $(-6x^2 - 3y^3 - 18z^4) + (-73x^2 + 6y^3 - 5z^4)$
10. $(8x + 5y^5 + 4z - 15) + (5x^2 + 8y^5 + 7z + 8)$
11. $(x^2 + 4y^3 - 6z^2) + (4x^2 + y^2 - z^2 + 6)$
12. $(-33a + 42b + 6c) + (-8a + 56b - 44c)$
13. $(16d^2 - 7e^2 - 16f) + (12d^2 + 7e^2 + 6f)$
14. $(27x^2 - 5y - 7z^2 + 60) + (-4x^2 + 5y - 6z^2 + 5)$
15. $(4x^2 + 5y^3 - 6z^2) + (2x^2 + 5y^2 - 45z^2)$
16. $(5x + 36y^3 - 3z^2) + (2x - y^3 + 59z^2)$
17. $(15a^3 + 2b^4 + 4c) + (-12a^3 + 8b^4 + 16c)$
18. $(-4r^2 - s - 7t^4) + (6r^2 - s - 53t^4)$
19. $(2x - 15y^3 + 6z^5 + 77) + (x + y^3 + z^5 + 2)$
20. $(-3c + b^2 - c^2 + 23) + (2c^2 + 5b^2 - 4c - 5)$
21. $(5p + 8q^3 - 3r^2) + (-75p + 6q^3 + 58r^2)$
22. $(x + y^2 + z^2 + 22) + (28x + y^2 + 13z^2 + 34)$
23. $(-x^2 - 27y^3 - 26z^2) + (-x^2 + 3y^3 - 38z^2)$
24. $(5x + 2y^5 + 23z - 55) + (x^2 + y^5 + 14z + 33)$
25. $(x^2 + 4y^3 - 25z^2) + (4x^2 + 4y^2 - z^2 + 14)$
26. $(-5a + 15b + 4c) + (-2a + 46b - 3c)$
27. $(8d^2 - 3e^2 - 2f) + (2d^2 + 4e^2 + 7f)$
28. $(6x^2 - 3y - 9z^2 + 8) + (-23x^2 + y - z^2 + 54)$
29. $(6x^2 + 3y^2 - 16z^2) + (34x^2 + 15y^2 - 23z^2)$
30. $(54x + 33y^3 - 2z^2) + (12x - y^3 + 4z^2)$
31. $(4a^3 + 6b^4 + 17c + 8) + (-6a^3 + 5b^4 + 14c)$
32. $(-5r^2 - s - t^4) + (4r^2 - 5s - 3t^4)$
33. $(2x - 65y^3 + 2z^5) + (x + 5y^3 + 29z^5 + 28)$
34. $(-13c + b^2 - 8c^2 + 9) + (c^2 + 15b^2 - 3c - 32)$
35. $(2p + q^3 - r^2) + (-6p + 5q^3 + 2r^2)$
36. $(x + y^2 + 12z^2 + 16) + (28x + 5y^2 + 7z^2 + 7)$
37. $(-x^2 - 27y^3 - 6z^2) + (-x^2 + 19y^3 - 8z^4)$
38. $(x + 5y^5 + z - 6) + (x^2 + 19y^5 + 13z + 22)$
39. $(x^2 + 24y^3 - 17z^2) + (18x^2 + 7y^2 - 8z^2 + 5)$
40. $(-73a + 2b + 27c) + (-28a + 26b - 4c)$
41. $(23d^2 - 9e^2 - 8f) + (3d^2 + 2e^2 + 6f)$
42. $(5x^2 - 29y - 6z^2 + 9) + (-x^2 + 27y - 6z^2 + 5)$
43. $(4x^2 + 5y^3 - 8z^2) + (4x^2 + 25y^2 - 34z^2)$
44. $(4x + 17y^3 - 3z^2) + (2x - y^3 + 13z^2)$
45. $(a^3 + 2b^4 + 4c) + (-5a^3 + 8b^4 + c)$
46. $(-3r^2 - 15s - 7t^4) + (6r^2 - s - 23t^4)$

SUBTRACTION OF POLYNOMIALS

To find the answer to a polynomial subtraction, turn the algebraic subtraction into algebraic addition and proceed like in addition. We do this by changing signs first.

Because polynomial subtractions must be enclosed in a parenthesis, this is done by changing **every** sign inside the polynomial.

Example: $(3x^3 + 2x^2 - 5x) - (5x^3 + x^2 - 2x)$

Remove parentheses. Notice how the whole trinomial in the second parenthesis changes signs

$$3x^3 + 2x^2 - 5x - 5x^3 - x^2 + 2x$$

Proceed with addition

$$\begin{array}{r} 3x^3 - 5x^3 = -2x^3 \\ 2x^2 - x^2 = x^2 \\ -5x + 2x = -3x \end{array} \rightarrow \text{Answer: } -2x^3 + x^2 - 3x$$

Example: $(-a^7 + 6a^4 - 4a^3) - (-5a^4 + 3a^3 - 8a^2)$

remove parentheses $-a^7 + 6a^4 - 4a^3 + 5a^4 - 3a^3 + 8a^2$

Combine coefficients according to base and exponent

$$\begin{array}{r} -a^7 \\ 6a^4 + 5a^4 = 11a^4 \\ -4a^3 - 3a^3 = -7a^3 \\ 8a^2 \end{array} \rightarrow \text{Answer: } -a^7 + 11a^4 - 7a^3 + 8a^2$$

Practice:

Subtract algebraically.

- $(3x^2 + 4y^2 - 6z^2) - (4x^2 + 5y^2 - 8z^2)$
- $(4x + 6y^3 - 8z^2) - (2x - y^3 + 9z^2)$
- $(13a^3 + 7b^4 + 10c + 7) - (-5a^3 + 8b^4 + 4c)$
- $(-9r^2 - 14s - 16t^4) - (14r^2 - 15s - 33t^4)$
- $(20x - 5y^3 + 22z^5) - (3x + 6y^3 + 9z^5 + 8)$
- $(-3c + 4b^2 - 6c^2 + 8) - (4c^2 + 5b^2 - 8c - 11)$
- $(12p + 14q^3 - 3r^2) - (-8p + 6q^3 + 21r^2)$
- $(x + y^2 + 11z^2 + 6) - (8x + 15y^2 + 5z^2 + 4)$
- $(-5x^2 - 7y^3 - 16z^4) - (-13x^2 + 9y^3 - 18z^4)$
- $(7x + 3y^5 + 7z - 5) - (2x^2 + 9y^5 + 3z + 2)$
- $(x^2 + 14y^3 - 16z^2) - (14x^2 + 15y^3 - 18z^2 + 8)$
- $(-13a + 12b + 7c) - (-18a + 6b - 14c)$
- $(10d^2 - 2e^2 - 6f) - (2d^2 + 8e^2 + 5f)$
- $(7x^2 - 9y - 5z^2 + 10) - (-3x^2 + 7y - 5z^2 + 9)$
- $(43x^2 + 54y^2 - 16z^2) - (24x^2 + 55y^2 - 44z^2)$
- $(54x + 16y^3 - 38z^2) - (22x - y^3 + 19z^2)$
- $(11a^3 + 27b^4 + 14c) - (-15a^3 + 81b^4 + 6c)$
- $(-44r^2 - 17s - 17t^4) - (64r^2 - s - 3t^4)$
- $(25x - 15y^3 + 2z^5 + 5) - (x + y^3 + z^5 + 22)$
- $(-32c + b^2 - c^2 + 8) - (24c^2 + 35b^2 - 47c - 1)$
- $(55p + 84q^3 - 37r^2) - (-77p + 9q^3 + 28r^2)$
- $(x + y^2 + z^2 + 9) - (8x + y^2 + 15z^2 + 14)$
- $(-x^2 - 57y^3 - 56z^2) - (-x^2 + 39y^3 - 28z^2)$
- $(97x + 23y^5 + 17z - 15) - (x^2 + y^5 + 13z + 12)$
- $(x^2 + 64y^2 - 5z^2) - (34x^2 + 45y^2 - z^2 + 4)$
- $(-63a + 14b + 43c) - (-28a + 36b - 33c)$
- $(18d^2 - 32e^2 - 26f) - (42d^2 + 48e^2 + 47f)$
- $(87x^2 - 39y - 3z^2 + 5) - (-13x^2 + y - z^2 + 19)$