Section 6.2
Polynomials

Polynomials are algebraic expressions with two or more terms separated by either an addition or a subtraction sign.

—A two-term polynomial is called a **binomial**
—A three-term polynomial is called a **trinomial**

Example: $3x^5 + 5x^2$ is a binomial $4y^3 + 2y^5 - y^6$ is a trinomial

We can write polynomials in descending order according to the value of the exponent

$$3x^7 + 2x^6 - 5x^5 + x^4 - 4x^3$$

Or in ascending order according to the value of the exponent

$$-4x^3 + x^4 - 5x^5 + 2x^6 + 3x^7$$

DEGREE OF THE POLYNOMIAL

The degree of the polynomial is established by the largest exponent found in any one term of the polynomial. If the term has more than one variable, we add the exponents of that term to determine the degree of the polynomial.

Example: The degree of the polynomial $x^6 + 5x^4 + 3x^2 + 7x$ is 6 because the largest exponent is 6.

Example: The degree of the polynomial $x^4y^3 + 3x^3y^3 + 2x^3y^2 + 4xy^2 + xy$ (7, 6, 5, 3, 2) is 7 because the highest sum [4 + 3 = 7] of the exponents of the polynomial is 7.

To raise an exponential expression to a power, multiply exponents

Example: $(x^a)^b = x^{a \times b}$ or $(x^2)^3 = x^{2 \times 3} = x^6$ or $(2^2)^3 = 2^{2 \times 3} = 2^6 = 64$

Example: $(3x^4)^3$ All the contents of the parenthesis are raised. $3 \cdot 3 \cdot 3x^{4 \times 3} = 27x^{12}$

NEGATIVE EXPONENTS AND ZERO EXPONENTS

Negative exponents, like positive exponents, may be written in descending order.

$$x^5 + x^4 + x^3 + x^2 + x + x^0 + x^{-1} + x^{-2} + x^{-3} + x^{-4} + x^{-5}$$

Because the division of two identical polynomials cancel each other, all polynomials to the zero (0) power are equal to 1.

$2 - 2 = 0 \rightarrow \frac{x^2}{x^2} = x^0 = 1$

Negative exponents are positive in the reciprocal. $x^{-4} = \frac{1}{x^4}$

Therefore, a descending list of monomials with negative exponents

$$x^5 + x^4 + x^3 + x^2 + x + x^0 + x^{-1} + x^{-2} + x^{-3} + x^{-4} + x^{-5}$$

may be written $x^5 + x^4 + x^3 + x^2 + x + x^0 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}$

Using negative exponents
Using the above expression and substituting 2 for x, the resulting polynomial would be:

$2^5 + 2^4 + 2^3 + 2^2 + 2 + 2^0 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5}$ **OR** $32 + 16 + 8 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$

Example: Multiply the monomials $(y^4)(2y^2)(4y^{-3})$

$4 + 2 - 3 = 3$ (exponents)

$(y^4)(2y^2)(4y^{-3}) = 8y^3$

$1 \times 2 \times 4 = 8$ (coefficients)

Example: Divide the monomials $\frac{x^5y^3z^{-2}}{x^2y^6z}$

$\frac{x^5y^3z^{-2}}{x^2y^6z} = x^3y^{-3}z^{-3}$

For x: $5 - 2 = 3$

For y: $3 - 6 = -3$

For z: $-2 - 1 = -3$

Answer: Make negative exponents positive $\frac{x^3}{y^3z^3}$

The answer may also be reached by solving negative exponents another way. In this way we first fix the negative exponent (making it positive in the reciprocal) and then reduce them like in a fraction.

$$\frac{x^5y^3z^{-2}}{x^2y^6z} = \frac{x^5y^3}{x^2y^6zz^2} = \frac{xxxxxyyy}{xxyyyyyyzzz} = \frac{xxx}{yyyzzz} = \frac{x^3}{y^3z^3}$$

Practice:

Name the polynomial

1. $5y^3 + 56y^2 - 4$
2. z
3. $345w^4x^5yz^2$
4. $123xyz - 543w^8x^5y^3z^2$
5. $w + x + 23y$
6. $(a + b)^2 + c$
7. $9x^2y^6$
8. $34a^4 + 234b^5 - 50a^6b^6c^9$
9. $p + 745p^2 - p^{23}$
10. $\frac{6}{7}a^3b^{14}c^9 + \frac{123abc}{21} - a$
11. $a + b + c + d + e + 2f$

Determine the degree of the polynomial

12. $4x^6 + x^5 + x^4 + x^3 + x^2$
13. $7x^2$
14. $6x^2y^2 - 5x^3y^5 - 3x^4y^7 - 4x^5y^3 - 8x^6y^4 - 5x^7y^6$
15. $5xy^3 - 4x^4y^6 - 2x^2y^3 - 9x^5y^6$
16. $2xy^6 - 3x^2y^3 - 4x^5y^3 - 3xy^8 - 6x^5y^3$
17. $7x^3y^7 - 9x^4y^5 - 2x^2y^3 - 3x^3y^8 - 6xy^7 - 7x^5y^4$
18. $3x^4y^3 - 8x^2y^4$
19. $9x^8y^3 - 3x^2y^3 - 5x^5y^6 - 8x^7y^8$
20. $x^3y - 2x^7y^6 - 9x^7y^9 - 3x^4y^{12} - 7xy^6$
21. $9x^3y^7 - x^2y^4 - 5x^9y^7 - 6x^2y^{13} - 3x^2y^3 - 2x^3y$
22. $x^3y^6 - 9x^4y^9 - 4x^6y^8 - 8x^3y^6 - x^4y^3$
23. $2x^7y^2 - 3x^4y^6 - 6x^2y^3$

Simplify

24. $\frac{4a^3b^{-2}c^{-1}}{2a^2b^4c^3}$
25. $\frac{18x^{-4}y^2}{9x^4y^{-2}}$
26. $\frac{8x^{-4}y^{-1}}{16x^3y^{-2}}$
27. $\frac{4a^{-6}b^{-7}}{12a^{-4}b}$
28. $\frac{4c^{-2}(d^2)^4}{20c^2d^{-2}}$
29. $\frac{10x^3y^{-2}}{30x^6y^{-7}}$
30. $\frac{6x^{-6}y^{-3}z^{-5}}{18x^{-4}y^{-3}}$
31. $\frac{18a^{-2}bc^{-5}}{36a^4b^{-2}c^{-1}}$
32. $\frac{36x^{-6}y^2z^{-8}}{4x^{-6}y}$
33. $\frac{12a^9b^6c^7}{4a^4b}$
34. $\frac{14a^7b^{-8}c^{-6}}{28(a^3)^3b^{-7}c^{-3}}$
35. $-\frac{21x^{-4}y^6z^{-8}}{3y^{-2}z^{-5}}$
36. $\frac{20x^{-5}y^{-7}z^{-3}}{4x^{-1}yz^{-3}}$
37. $-\frac{22x^{-4}y^{-3}z^{-11}}{11x^{-6}yz^{-14}}$
38. $\frac{5(a^4)^{10}b^{-2}c^{-4}}{10a^2b^{-5}c^{-4}}$
39. $\frac{7m^{-22}n^{-24}p^{-9}}{21m^{-25}n^{-23}p^{-16}}$
40. $\frac{18x^{-3}y^{-1}}{27x^3y^{-2}}$
41. $-\frac{9x^{-6}y^{-7}}{54x^5y^{-6}}$
42. $\frac{-4a^{-3}b^{-2}}{-32a^{-5}b^{-1}}$
43. $\frac{6c^{-7}(d^3)^5}{18c^{-8}d^{-3}}$
44. $\frac{5x^{-3}y^{-7}}{30x^{-6}y^{-3}}$
45. $\frac{8x^{-2}y^{-8}z^{-7}}{36x^{-8}y^4}$
46. $\frac{14x^6y^8z^4}{8xyz^{10}}$
47. $-\frac{48x^3yz^{12}}{30x^5yz^9}$
48. $\frac{6(a^3)^8b^5c^7}{24a^2b^9c^5}$
49. $\frac{14m^{14}n^{-2}p^{-19}}{49m^{-2}n^{-2}p^{-1}}$
50. $\frac{25x^3y^2}{5x^5y^{-8}}$
51. $-\frac{2x^6y^8}{16x^3y^{-5}}$
52. $\frac{-6a^{-7}b^{-8}}{-42a^{-1}b^{-5}}$