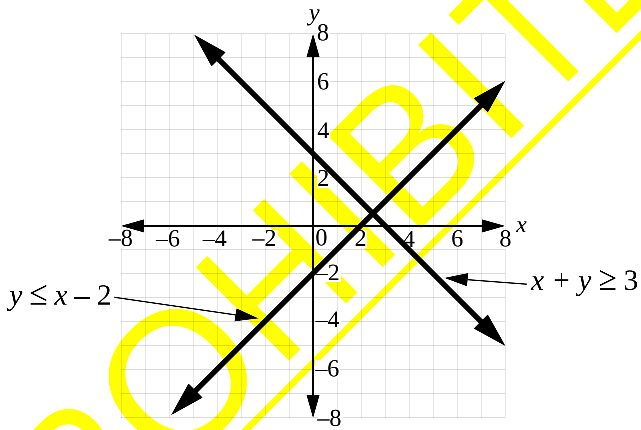
Section 5.5
Graphing Systems of Inequalities

The solution to a system of equations is the point where the lines meet. The solution to a system of inequalities is not a point but a region surrounded by the lines the system of inequalities represents.

Example: Find the solution to the inequalities $x + y \geq 3$ and $y \leq x - 2$

First, plot the inequalities using the same method used for equations:



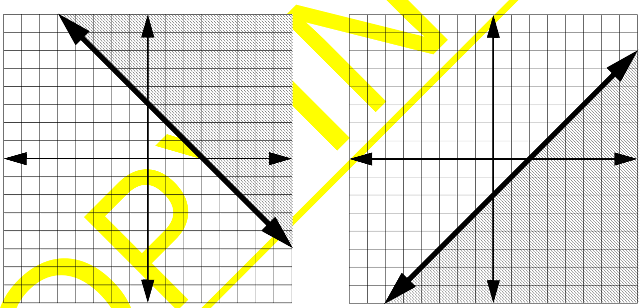
The slope and y-intercept of the first inequality:

$$x + y \geq 3$$
$$x - x + y \geq -x + 3$$
$$y \geq -x + 3$$

The slope (coefficient of x) is –1 and the y-intercept is 3.

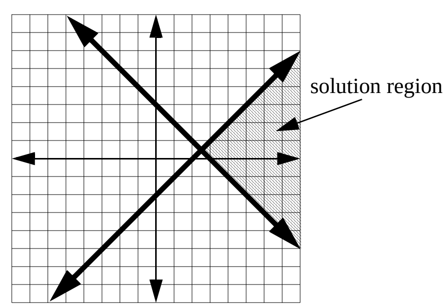
The slope of the second inequality is 1 (coefficient of x) and the y-intercept is –2. Both lines are plotted.

To find the region of the answer, look at the direction of the y inequalities only. In the first inequality, y is "greater than"; therefore, the solution is *above* (greater). The second inequality is "less than", thus the solution must be *below* the line. The graphs below show this for each line separately and then a third graph shows the solution (both combined).



$y \geq -x + 3$ $\qquad$ $y \leq x - 2$

Plotted separately

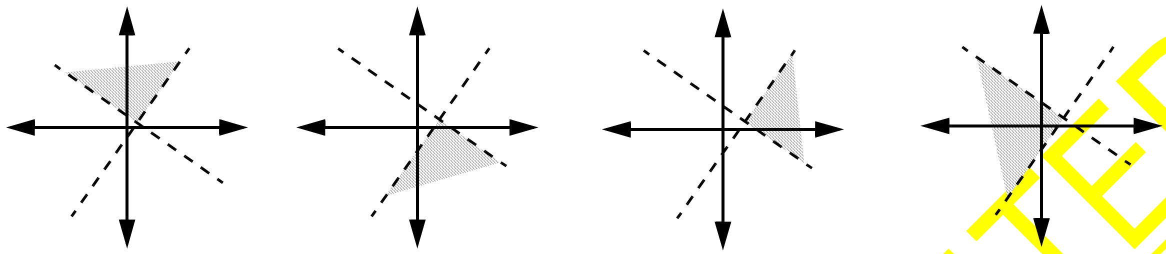


Solution region for both inequalities together. Every point in the solution region will work for both inequalities.

Solution regions for pairs of inequalities (positive slope and negative slope):

Examples:

y >	y <	y >	y <
y >	y <	y <	y >



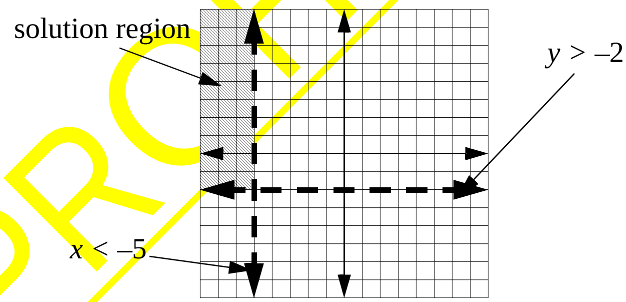
Both pos. and neg. slopes "greater than"	Both pos. and neg. slopes "less than"	Neg. slope "greater than" pos. slope "less than"	Pos. slope "greater than" neg. slope "less than"

Example: Find the solution region for $x < -5$ and $y > -2$

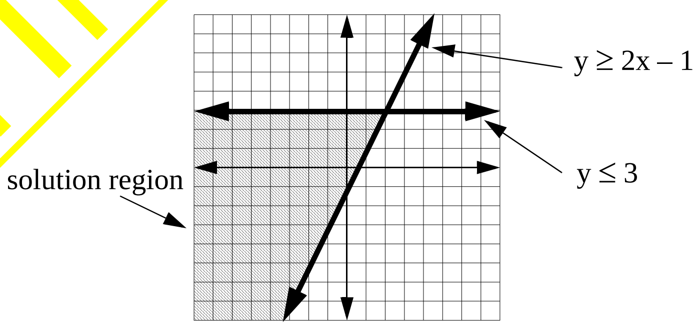
The first inequality is to the left of a vertical line
(x is "less" to the left.)

The second inequality is above a horizontal line
(y is "greater" over the horizontal.)

Because the inequalities are < and >, dashed lines apply.



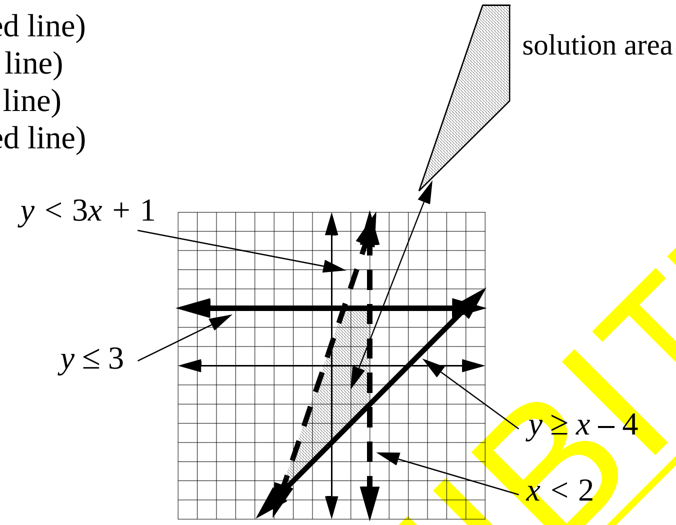
Example: Find the solution region for $y \le 3$ and $y \ge 2x - 1$



Because both lines are ≤ and ≥, solid lines apply.

Example: Find the solution area to the four inequalities:

$x < 2$ (dashed line)
$y \le 3$ (solid line)
$y \ge x - 4$ (solid line)
$y < 3x + 1$ (dashed line)



Practice:
Find the solution area.

1. $y \ge x + 5$
   $y \ge -x + 3$
2. $x + y > 8$
   $x - y > 6$
3. $x + y < 1$
   $y + 4x < 6$
4. $y < -x - 2$
   $3x + y \le -5$
5. $3y \ge 2x + 12$
   $-5x + 6y + 14 \le 0$
6. $4y > 5x + 8$
   $-3x + 5y \le 2$
7. $-5x < 3y - 15$
   $2y < 3x - 4$
8. $5x + 4y + 12 \le 0$
   $4y - 3x + 8 \le 0$
9. $4y \ge 7x + 11$
   $4y \ge -2x - 9$
10. $8x > 7y - 12$
    $3x + y + 13 > 0$
11. $3y \le 9x + 13.5$
    $y < -7$
12. $5y \le 2x - 8$
    $7x + 6y \le -6$
13. $3y \le 4x + 6$
    $-2x + 3y + 15 \ge 0$
14. $7x \le 4y + 1$
    $4x + 2y > 15$
15. $-9x > -4y - 16$
    $4y < 9x - 8$
16. $6x + 3y + 17 \le 0$
    $8y + 5x + 14 \le 0$
17. $4y \ge 9x + 24$
    $3y \ge -5x + 15$
18. $5y > 7x - 21$
    $7x + 5y + 10 > 0$
19. $2y < 7x + 11$
    $y \le 5$
20. $3y \le 8x - 12$
    $3x + 5y \le 12$
21. $2y \ge 3x$
    $-2x + 3y - 6 \le 0$
    $y \ge x - 1$
22. $3y > 7x + 12$
    $3x + 4y \le 9$
    $y < -x + 3$
23. $-3x > -3y - 6$
    $y < 5x - 5$
    $x \ge -1.5$
24. $5x + 2y + 7 \ge 0$
    $7y + 5x + 14 \ge 0$
    $y \le 3x - 5$
25. $2y \le 3x + 4$
    $4y \le -3x + 3$
    $2y < x + 1$
26. $2y > 5x - 7$
    $x + 2y + 6 > 0$
    $3y < -x + 4$
27. $3y < 5x + 8$
    $y \ge -6$
    $x \le 3$
    $3y < 2x + 1$