Section 5.4
Systems of Equations and Problem Solving

Making quantitative decisions means that, after comparing numbers, we sometimes select the lowest price or perhaps the highest quantity, the fastest time, or maybe the shortest distance. But if we want the shortest time (for example, a trip) at the lowest price, could that be a problem? Sometimes selecting alternatives that are hard to match IS the problem. Setting up systems of equations that represent conflicting decision-making could go a long way in helping us solve quantitative problems.

Example:
Using the first example from section 3.6 and expanding it, we see that Pat needs to rent a truck for a few hours to move. However, this time she will have two competing alternatives, the original one where she would pay $40 per day and $0.50 per mile, and a new one where she only pays $35 per day, but the cost per mile increases to $0.54 per mile. Which one will it be, lower daily rate, or lower per mile rate?
Obviously the answer rests on the number of miles she plans to drive: Select lower "per day" alternative if the trip is short, or lower "per mile" alternative, if the trip is long. In any case, we must find that "miles" number where the total costs of both alternatives are the same.
We could solve the problem by trial and error, or we can write two linear equations that represent the conflicting alternatives and do it faster.

Alternative 1: \$0.50 per mile plus \$40 per day → Cost = $0.50M + 40$ where M is miles
Alternative 2: \$0.54 per mile plus \$35 per day → Cost = $0.54M + 35$

When costs are equal, what is the value of M?

$$0.50M + 40 = 0.54M + 35$$
$$0.50M - 0.54M = 35 - 40$$
$$-0.04M = -5$$
$$M = \frac{5}{0.04} = 125 \text{ miles}$$

Select alternative 1 if the trip is more than 125 miles and alternative 2 if the trip is less than 125 miles

Example:
Issac is 6 years older than Jose. Issac was three times as old as Jose two years ago. What are their ages now?

Two alternatives, two equations: One today, the other one two years ago.

Today: $I = J + 6$

Two years ago: $I - 2 = 3(J - 2)$
$I = 3(J - 2) + 2$
$I = 3J - 6 + 2$
$I = 3J - 4$

Solve equation for today and equation for two years ago as a system:

$I = J + 6$
$I = 3J - 4$
$3J - 4 = J + 6$
$3J - J = 6 + 4$
$2J = 10$
$J = \frac{10}{2} = 5$

If today Jose = 5, then Issac = 5 + 6 = 11

Example:
Two cars start a trip toward each other at the same time. One from Los Angeles, CA, to Miami, FL, another one from Miami to Los Angeles. The Miami car travels at 48 miles per hour, and the Los Angeles car at

52 miles per hour (average speed). If the distance between the two cities is 2,735 miles, how long will it take for the two cars to meet?

Because distance is a function of speed and time ($d = st$) and here time (t) is the same for both cars, solve for t.

$$t_{Miami} = \frac{d_M}{s_M} \quad , \quad t_{Los\ Angeles} = \frac{d_{LA}}{s_{LA}} \quad \text{and} \quad t_{Miami} = t_{LA} \quad \text{therefore} \rightarrow \frac{d_{LA}}{s_{LA}} = \frac{d_M}{s_M} \quad (1)$$

Because $d_{LA} + d_M = 2735$ then solve for d_M and substitute $d_M = 2735 - d_{LA}$ in equation (1)

Solving equations as a system $\frac{d_{LA}}{s_{LA}} = \frac{2735 - d_{LA}}{s_M}$ and substituting for s_{LA} and s_M $\frac{d_{LA}}{52} = \frac{2735 - d_{LA}}{48}$

Now cross-multiply and solve for d_{LA}:

$$\begin{aligned} 48d_{LA} &= 52(2735 - d_{LA}) \\ 48d_{LA} &= 142220 - 52d_{LA} \\ 52d_{LA} + 48d_{LA} &= 142220 \\ 100d_{LA} &= 142220 \\ d_{LA} &= \frac{142220}{100} = 1{,}422.2 \text{ miles} \end{aligned}$$

If d_{LA} = 1,422.2 miles,
then $d_M = 2735 - 1422.2 = 1312.8$ miles

They would meet at 1,422.2 miles from Los Angeles, or 1,312.8 miles from Miami.

Time: $t_{LA} = \frac{1422.2}{52} = 27.35$ hours

Note on the use of "subscripts."
The problem above uses "subscripts", which are small letters placed slightly below a letter or number (d_{LA}). This letters have no numerical value and are there to clarify the name of a variable using the same letter, for example "time for Miami" is t_M and "time for Los Angeles" is t_{LA}. The same may be done for distance (d_M, d_{LA}) and speed (s_M,

Example:
In a basketball game at school, 620 fans paid admission. If a student ticket was $3 and general admission ticket $8, how many students attended if $2,405 were collected?

Two equations must be extracted from the information:

Students + General = 620 or S + G = 620 (1)
Students(3) + General(8) = 2405 or 3S + 8G = 2405 (2)

Solving system by algebraic addition:
Multiplying (1) by –3

$$\begin{array}{r} 3S + 8G = 2405 \\ -3S - 3G = -1860 \\ \hline 5G = 545 \end{array}$$

$G = \frac{545}{5} = 109$ fans, general admission

620 – 109 = 511 students

Practice:

1. At the produce store, Gerri bought 18 mangoes and avocados. The typical mango weighs 9 ounces and the typical avocado 12 ounces. If the total weight of the fruit was 195 ounces, how many of each fruit did she buy?
2. A piggy bank has $21.70 in quarter and dimes. If there are 160 coins, how many are quarters and how many are dimes?

3. A car rental company has two options for compact cars, a gas-powered one for $34 per day and $0.18 per mile, and a hybrid one for $25 a day and $0.28 per mile. How many miles must you drive to make the gas-powered one the better choice?
4. The total paid admission at a professional baseball game was $1,465,000 with 33,000 fans attending. If the upper deck seats cost $35 and the lower deck $55, how many fans bought upper deck tickets?
5. Shakira is 11 years older than her sister. Three years from now, she will be twice her sister's age. What are the ages of the sisters today?
6. Two cars start a trip in opposite directions at the same time and stop at the same time. Car one travels at 55 miles per hour and car two at 65 miles per hour. If when they stop the distance between them is 780 miles, how long did they travel?
7. Today, Lauren is seven years older than her brother. Five years from now she will be twice his age. How old are both siblings now?
8. A rectangle is 4.8 feet longer than it is wide. If the perimeter of the rectangle is 32.4 feet, find the width and length of the rectangle.
9. Two angles are supplementary (two angles that add up to 180º). If the difference between the angles is 44º, find the angles.
10. A boat travels down a river at 22 miles per hour to reach its destination in 2.5 hours. Coming back, the trip took 5.5 hours. Find the speed of the river's current.
11. A concert hall sold 1750 tickets. Tickets are $75 for section A and $45 for section B. If the total gate was $97,410, how many tickets were sold for section A, how many for section B?
12. A train leaves Paris traveling at 120 kilometers per hour (kph). Three hours later, and going in the same direction, a "bullet" train leaves the same station while traveling at 220 kph. How long will it take for the bullet train to catch the first train?
13. A church collection basket held $965. If the basket consisted of 141 bills of $5s and $10s, how many of each denomination were in the basket?
14. A propeller plane takes four hours longer than a jet plane to complete the same trip between two cities. If the jet plane travels at 513 miles per hour (mph) and the propeller plane at 285 mph, how far are the two cities?
15. A truck rental company charges $1.07 per mile for truck A and $1.23 per mile for truck B. If the daily rate for truck A is $84 and the daily rate for truck B is $72, how many miles must be driven to make the cost of both trucks the same?
16. A professional baseball team is giving away banners and caps to the first 8,000 fans to enter the stadium on Sunday. If each caps is $3.50, and each banner $2.10, and the team spent $21,700 in the promotion, how many caps and how banners were given away?
17. An airplane flew from Boston to Miami in 2.4 hours with a tail wind of 75 miles per hour. The trip from Miami to Boston took 3.1 hours against the same wind. Find the speed of the airplane assuming there is no wind.
18. Larrisha deposited 125 bills in the bank in $20 and $50 denominations. If she gave the bank's teller $3,490, how many bills were $20, how many $50?
19. Samuel was 15 years younger than Mark forty-five years ago. Today he is 80% Mark's age. How old are they now?
20. Brittany bought some blue and some white envelopes at Officeland. White envelopes cost $0.02 per unit and blue ones $0.035 per unit. If the total cost of the envelopes is $240.50, how many are white and how many are blue?