Section 5.3
Systems of Equations: Solve by Addition

THE ADDITION METHOD

The "addition" method is actually the "elimination by algebraic addition" method. Where, instead of adding all the time, we also have to use the number-line approach of eliminating negatives with positives. The object of the approach is to either eliminate x to find y, or eliminate y to find x.

Example: Solve the following system by addition.

$$\begin{aligned} x - y &= 5 \\ x + y &= 8 \end{aligned}$$

Second column $-y + y = 0$ (the ys are eliminated)

First column, $x + x = 2x$

$$\begin{array}{rl} x - y &= 5 \\ x + y &= 8 \\ \hline 2x &= 13 \end{array}$$

Third column $5 + 8 = 13$

$$x = \frac{13}{2}$$

Divide both sides by 2

$$x = 6.5$$

As before, to find the value of y, substitute x into any of the two equations:

$$\begin{aligned} x + y &= 8 \\ 6.5 + y &= 8 \\ 6.5 - 6.5 + y &= 8 - 6.5 \\ y &= 8 - 6.5 \\ y &= 1.5 \end{aligned}$$

The solution to the system is point (6.5,1.5)

Example: Solve the following system by addition.

$$\begin{aligned} 2x + y &= 5 \\ x + y &= 3 \end{aligned}$$

If we add the columns the way they are, no elimination will take place; therefore, to eliminate the y variable, the second equation will change to $-x - y = -3$. We achieve this by multiplying both sides of the second equation by -1.

$$-1(x + y = 3) \longrightarrow -x - y = -3$$

Now add equations:

$$\begin{array}{rl} 2x + y &= 5 \\ -x - y &= -3 \\ \hline x &= 2 \end{array}$$

If $x = 2$, then

$$\begin{aligned} 2 + y &= 3 \\ y &= 3 - 2 \\ y &= 1 \end{aligned}$$

The solution to the system is point (2,1).

Example: Solve the following system by addition. $5y = 12 - 2x$, $3x + 4y = 4$

First, rearrange equation and place like terms in line (x above x and y above y) $2x + 5y = 12$, $3x + 4y = 4$

This time the x will be eliminated by multiplying the first equation by 3 and the second equation by –2.

$$\begin{aligned} 3(2x + 5y &= 12) \\ -2(3x + 4y &= 4) \end{aligned} \quad \rightarrow \quad \begin{aligned} 6x + 15y &= 36 \\ -6x - 8y &= -8 \\ \hline 7y &= 28 \end{aligned}$$

$$y = \frac{28}{7}$$

$$y = 4$$

Substituting 4 for y in the second equation:

$$\begin{aligned} 3x + 4(4) &= 4 \\ 3x + 16 &= 4 \\ 3x + 16 - 16 &= 4 - 16 \\ 3x &= -12 \\ x &= -\frac{12}{3} \\ x &= -4 \end{aligned}$$

The solution to the system is point (–4,4)

Practice:
Solve each system by algebraic addition.

1. $y = x + 2$, $y = -x + 4$
2. $x + y = 7$, $x - y = 5$
3. $x + y = 3$, $y + 2x = 5$
4. $y = -3x - 4$, $2x + y = -6$
5. $2y = 3x + 10$, $-3x + 5y + 11 = 0$
6. $5y = 3x + 9$, $-2x + 4y = 1$
7. $-6x = -2y - 10$, $3y = 4x - 5$
8. $4x + 3y + 11 = 0$, $5y - 4x + 9 = 0$
9. $5y = 8x + 12$, $5y = -3x - 10$
10. $7x = 5y - 33$, $2x + y + 16 = 0$
11. $8y = 2x + 11$, $y = -8$
12. $7y = 3x - 9$, $2x + 5y = -5$
13. $4y = 3x + 13$, $-3x + 4y + 17 = 0$
14. $6x = 3y + 15$, $3x + 3y = 14$
15. $-7x = -5y - 16$, $3y = 7x - 16$
16. $7x + 5y + 15 = 0$, $9y + 7x + 19 = 0$
17. $12y = 12x + 23$, $4y = -6x + 17$
18. $13y = 14x - 14$, $28x + 6y + 16 = 0$
19. $2y = 9x + 19$, $y = 27$
20. $3y = 15x - 11$, $3x + 4y = 15$
21. $5y = 2x + 17$, $-3x + 4y + 13 = 0$
22. $8y = 7x + 5$, $3x + 4y = 19$
23. $-7x = -3y - 16$, $y = 5x - 12$
24. $8x + 6y + 17 = 0$, $7y + 5x + 15 = 0$
25. $5y = 9x + 12$, $4y = -3x + 13$
26. $2y = 5x - 17$, $9x + 12y + 66 = 0$
27. $13y = 15x + 18$, $y = -15$