Section 5.2
Systems of Equations: Solve by Substitution

THE SUBSTITUTION METHOD

Using the second example from section 5.1: $y = 2x - 1$ $\qquad y = \frac{2}{3}x + 2$

We read these two equations and realize that the same variable, *y*, is equal to two distinct expressions.

because $\quad y = y$

then $\quad 2x - 1 = \frac{2}{3}x + 2$

When we pair these two expressions, we get rid of *y* and are left with one *x* on each side. We then combine "like terms" and solve for *x*.

Solving a rational equation (an equation with fractions) is easier if we eliminate the fractions and turn them into integers. We do this by multiplying each term of the equation by 3 (because the 3 denominator is what makes it a fraction).

$$(3)2x - (3)1 = (3)\frac{2}{3}x + (3)2$$

The result is that we exchange cancelling the 3 denominator for a larger equation—of the same value—overall. The new equation is now:

$$6x - 3 = 2x + 6$$

$6x - 2x = 6 + 3$ $\qquad$ Combine like terms

$4x = 9$ $\qquad$ Divide both sides by 4

$$x = \frac{9}{4} = 2.25$$

The graphical solution in section 5.1 estimated $x = 2.3$, but $\mathbf{x = 2.25}$ is accurate and exact.

To find the *y* value of the equation, go back to either of the two original equations, substitute the value of *x*, and get the value for *y*.

$$y = 2(2.25) - 1$$
$$y = 4.5 - 1$$
$$y = 3.5$$

The point where the lines cross is (2.25, 3.5). Any system of equations can be solved by substitution.

Example: Solve the system by substitution $\qquad y = 2x + 7$

$y = 2x + 4$ $\qquad$ Because $y = y$, substitute

$$2x + 7 = 2x + 4$$
$$2x - 2x = -7 + 4$$
$$0 \neq -3$$

Because 0 is not equal to –3, the lines will not meet and there is no solution to the system: The lines are parallel. If the solution had been a true statement, like –3 = –3, then there is only one solution (all the points are at the intersection) and both lines are the same (identity property).

Example: Solve the system by substitution

$$2x - 3y = 10$$
$$x + y = 2$$

To substitute, first solve one of the equations in terms of x or y. Solving for x, the second equation becomes:

$$x = -y + 2$$

Substituting $(-y + 2)$ for x into the first equation: $2(-y + 2) - 3y = 10$

Doing this gives us an equation without x.

Solving for y:

$2(-y + 2) - 3y = 10$	Multiply contents of parenthesis by 2
$-2y + 4 - 3y = 10$	Combine like terms
$-5y + 4 = 10$	Subtract 4 from both sides
$-5y + 4 - 4 = 10 - 4$	
$-5y = 6$	Divide both sides by -5
$y = \frac{6}{-5} = -1.2$	

Because $x = -y + 2$ and $y = -1.2$,

by substitution, then

$$x = -(-1.2) + 2$$
$$x = 1.2 + 2$$
$$x = 3.2$$

The solution to the system is point (3.2, –1.2)

Practice:
Solve each system by substitution.

1. $2y = 3x + 5$
   $y = x + 3$
2. $y = 4x - 5$
   $x + 2y + 6 = 0$
3. $3y = 5x$
   $y = -5$
4. $y = -x - 7$
   $x + y = -3$
5. $y = x + 5$
   $-x - y + 3 = 0$
6. $y = x + 6$
   $-6x + y = 3$
7. $-2x = -3y - 7$
   $y = x - 1$
8. $x + y + 2 = 0$
   $y - x + 7 = 0$
9. $y = 3x + 1$
   $y = -x + 6$
10. $x = 2y - 3$
    $3x + y + 5 = 0$
11. $y = 2x + 3$
    $y = 6$
12. $7y = x - 10$
    $x + 4y = -12$
13. $5y = x + 1$
    $-x + 4y + 7 = 0$
14. $x = y + 13$
    $3x + y = 4$
15. $-2x = -y - 6$
    $y = x - 5$
16. $3x + 2y + 5 = 0$
    $y + 2x + 9 = 0$
17. $2y = 2x + 10$
    $y = -5x + 11$
18. $3y = 4x - 4$
    $x + y + 6 = 0$
19. $6y = x + 9$
    $y = 7$
20. $y = 5x - 1$
    $2x + 3y = 5$
21. $3y = 5x + 7$
    $-3x + 9y + 3 = 0$
22. $3y = x + 15$
    $3x + y = 9$
23. $-4x = -2y - 5$
    $y = 3x - 2$
24. $2x + 3y + 7 = 0$
    $3y + x + 5 = 0$