Section 5.1
Systems of Equations: Solve by Graphing

Systems of equations refer to the solution of two or more equations.

Because equations are represented by lines and these lines will eventually cross at a point (unless they are parallel), the solution to the system takes place when we find the point where the lines join.

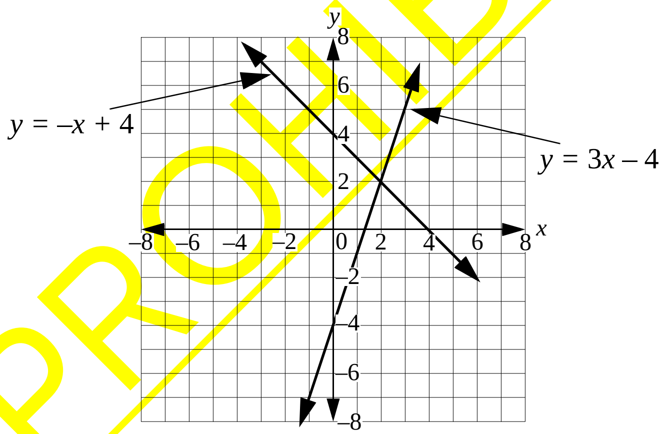
The most logical way to find the solution of a system is to plot the equations and read from the graph the coordinates (point) where the lines meet.

GRAPHICAL SOLUTION

Find the solution to the system of equations represented by $y = 3x - 4$ and $y = -x + 4$

First plot $y = 3x - 4$
where the y-intercept is (0,–4) and the slope (the coefficient of x) 3.

Second, plot $y = -x + 4$
where the y-intercept is (0,4) and the slope –1.



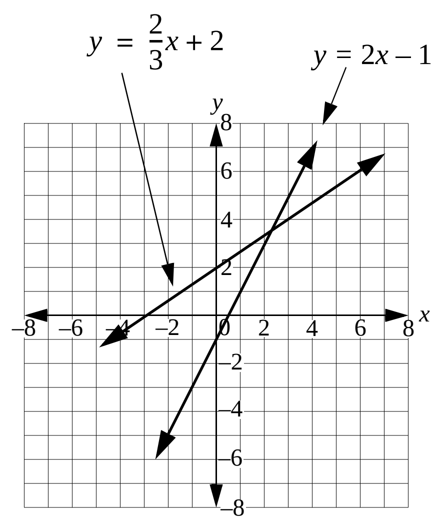
In the graph the lines cross at (2,2). This point is the solution to the system represented by the two equations given.

Graphical solutions have the drawback that the accuracy of the answer depends on how well you can plot the graph. Some systems may not be as simple as the one above.

Example: Find the solution to the system

$$y = 2x - 1$$
$$y = \frac{2}{3}x + 2$$

Plot the y-intercept and slope. The y-intercept for the first equation is at (0,–1) and the slope is 2. The y-intercept for the second equation is at (0,2) and the slope is $\frac{2}{3}$.



The graph shows the system plotted and the lines form a point at, approximately, (2.3, 3.6). Notice that it is difficult to define points that fall between integers, where exact answers are elusive.

To solve this dilemma, there are two other methods that may be used. One method is based on substituting one equation into the other, and the second method involves algebraic addition and substitution. Both are discussed in the next two sections, 5.2 and 5.3.

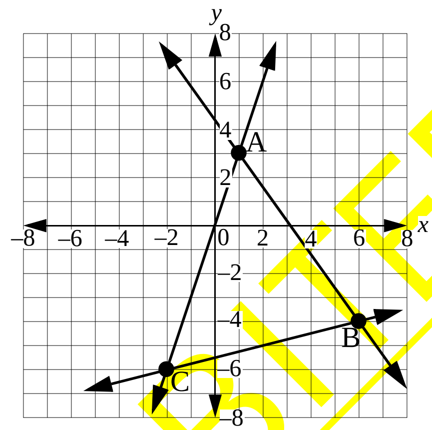
Example: Find the equations of the three lines that intersect at A(1,3), B(6,–4), C(–2,–6) to form triangle ABC.

Line AB

slope: $m = \dfrac{change\ \ of\ \ y_{AB}}{change\ \ of\ \ x_{AB}} = \dfrac{-7}{5}$ y-intercept: (0,4.3)

$y = mx + b$

$y = \dfrac{-7}{5}x + 4.3$



Line CB

slope: $m = \dfrac{change\ \ of\ \ y_{CB}}{change\ \ of\ \ x_{CB}} = \dfrac{2}{8} = \dfrac{1}{4}$ y-intercept: (0,–5.5)

$y = \dfrac{1}{4}x - 5.5$

Line CA

slope: $m = \dfrac{change\ \ of\ \ y_{CA}}{change\ \ of\ \ x_{CA}} = \dfrac{9}{3} = \dfrac{3}{1} = 3$ y-intercept: (0,0) is an empty value.

$y = 3x$ (empty)

Practice:

Solve systems graphically.

1. $y = x + 2$
   $y = -x + 6$
2. $y = 2x - 5$
   $x + y + 2 = 0$
3. $y = 3x + 5$
   $y = 2$
4. $3y = 2x - 9$
   $x + 3y = -18$
5. $2y = x + 2$
   $-3x + 2y + 2 = 0$
6. $y = x + 11$
   $2x + y = 3$
7. $-3x = -2y - 4$
   $y = x - 3$
8. $5x + 4y + 4 = 0$
   $2y + x + 8 = 0$

Solve systems graphically.
Name figure formed.

9. $y = x + 4$
   $y = -x + 4$
   $y = -2$
10. $x = -4$
    $x + y = 1$
    $y = 2x - 2$
11. $3y = 2x + 6$
    $y = -3$
    $3x + 2y = 0$
12. $x = -3$
    $y = -3$
    $x + y - 2 = 0$
13. $y = 6$
    $y = 5$
    $x = 2$
    $x = -4$
14. $y + x = 6$
    $y = x - 2$
    $y = -x - 4$
    $y = x + 2$
15. $y = x + 5$
    $y + x = 5$
    $y = x - 5$
    $y + x + 5 = 0$
16. $x = -4$
    $x = 2$
    $y = -2x + 2$
    $y = 2x - 4$
17. $2x + y = 2$
    $y = 2x - 2$
    $-2x + y - 6 = 0$
    $2x + y = -6$