Section 4.2
Absolute Values

A value becomes absolute if we decide to disregard the opposite (negative) nature of numbers. In practice all absolute values become positive.

Absolute values are always shown surrounded by two vertical bars.

Examples:

1. $|9| = 9$
2. $|-9| = 9$
3. $|-14 + 6| = |-8| = 8$
4. $|-7 + 23| = |16| = 16$
5. $-18 + |-5 + 2| = -18 + |-3| = -18 + 3 = -15$

NOTE: (In 5 above, because –18 is not absolute—no bars—the answer is negative. Only –3 turns positive).

EQUATIONS WITH ABSOLUTE VALUES

Because the answer to an absolute expression is never negative, we then must realize that each unknown absolute value has the possibility of coming from either a positive or a negative value, producing then two solutions.

Example: In $|x| = 8$ the value for x could be 8 or –8

One possible answer is positive and one is negative.

Example:

$$|y| + 9 = 14$$
$$|y| = 14 - 9$$
$$|y| = 5$$
$$y = 5 \qquad y = -5$$

Example: $|5x - 7| = 8$

positive outcome

$$|5x - 7| = 8$$
$$5x - 7 = 8$$
$$5x = 8 + 7$$
$$5x = 15$$
$$x = 3$$

or

negative outcome

$$|5x - 7| = -8$$
$$5x - 7 = -8$$
$$5x = -8 + 7$$
$$5x = -1$$
$$x = \frac{-1}{5}$$

ABSOLUTE EQUATIONS EQUAL TO NEGATIVE OUTCOMES CANNOT BE SOLVED.

Examples:

1. $|5x - 7| = -8$ (No solution. Absolute value cannot be negative).
2. $|5x - 7| + 10 = 8$ (Right side becomes $-10 + 8 = -2$. No solution).

Practice:
Solve.

1. $|x| - 4 = 12$
2. $|y| + 1 = 7$
3. $|2a + 3| = 7$
4. $2 = -4 + |s|$
5. $3 + |x| = 9$
6. $7 = |b| + 8$
7. $|x - 17| = -8$
8. $|y| + 6 = 7$
9. $|3a| + 8 = 56$
10. $16 = 23 + |s|$
11. $|2 + x| = 14$
12. $4 = -|b + 6|$
13. $|x| - 5 = 50$
14. $|y + 9| = 28$
15. $22 = |4a + 8|$
16. $-5 = |14 + s|$
17. $|42 + x| + 8 = 2$
18. $19 = |b + 4|$
19. $|x| - 23 = -6$
20. $|y + 2| = 16$
21. $|2b| + 13 = +5$
22. $-28 = |33 + t|$
23. $17 - |y| = 29$
24. $72 = |b + 8|$
25. $|y - 13| = 72$
26. $|u + 12| = 22$
27. $|2c| + 21 = 6$
28. $39 = 21 + |p|$
29. $40 - |g| = 9$
30. $-22 = -|v + 4|$
31. $|f - 1| = 24$
32. $|h + 16| = 42$
33. $65 = |t + 18|$
34. $1 = 1 + |s|$
35. $|15 - x| + 39 = 37$
36. $11 = -|c + 55|$
37. $|x - 3| - 8 = 5$
38. $|y + 8| = 13$
39. $28 = |4a| + 1$
40. $12 = |4 + u|$
41. $|18 - x| = 3$
42. $14 = |b + 9|$
43. $|x - 9| = 23$
44. $|y| + 22 = 14$
45. $|3a + 4| = 6$
46. $56 = |p|$
47. $|23 - x| = 45$
48. $43 = |b| + 24$
49. $|x - 15| = 32$
50. $|y| + 7 = 54$
51. $-33 = |7a| + 87$
52. $67 = |24 + a|$
53. $|19 - x| = 54$
54. $3 = -|b + 44|$
55. $|x - 18| = 8$
56. $|y + 17| = 83$
57. $|y| - 71 = -23$
58. $51 = |4 + t|$
59. $|7 + d| = 72$
60. $95 = |g + 42|$
61. $|x - 55| = 77$
62. $|r + 72| = 28$
63. $6 = |d| + 22$
64. $43 = 54 + |u|$
65. $|43 - s| + 4 = 66$
66. $32 = -|p + 65|$
67. $|x - 5| = 7$
68. $|u + 8| = 95$
69. $2|h| + 55 = 5$
70. $31 = 6 + |s|$
71. $|32 - x| = 84$
72. $15 = |b + 18|$
73. $|x - 25| = 66$
74. $|y| + 8 = 6$
75. $11 = |c + 44|$
76. $53 = 44 + |s|$
77. $|57 + x| = 39$
78. $8 = |k + 3|$
79. $|x| - 3 = 7$
80. $|k + 9| = 6$
81. $|a| + 8 = 5$
82. $43 = |91 + m|$
83. $|31 + x| = 8$
84. $11 = |b + 8|$
85. $|x - 3| = 5$
86. $|y - 3| = 63$
87. $9 = |a + 9|$
88. $35 = |7 + b|$
89. $|9 - x| = 43$
90. $4 = |b + 5|$
91. $|f - 5| = 2$
92. $|y + 3| = 10$
93. $2 = 3|h + 4|$
94. $13 = |5 + h|$
95. $|5 + x| = 8$
96. $33 = |b + 9|$
97. $|x - 35| = 44$
98. $|2y| + 14 = 12$
99. $|x - 13| = 20$

INEQUALITIES WITH ABSOLUTE VALUES

Inequalities give a "range" of values. When inequalities include absolute expressions,

a **conjunction** ($|x| < 5$) or a **disjunction** ($|x| > 4$) is formed.

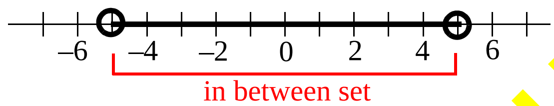
In a **conjunction** (sometimes defined as "in between sets"), two values emerge. They become external limits (for example, a fixed set of integers) beyond which answers will not be found. In the conjunction

$$|x| < 5$$

the answer yields two boundaries $x < 5$ and $x > -5$ (less than "+", greater than "–")

or $-5 < x < 5$

graphing it

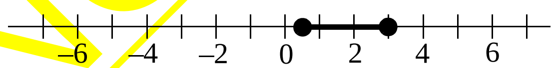


Example: $|4x - 7| \leq 5$

positive possibility	negative possibility
$4x - 7 \leq 5$	$4x - 7 \geq -5$
$4x \leq 7 + 5$	$4x \geq 7 - 5$
$4x \leq 12$	$4x \geq 2$
$x \leq \frac{12}{4}$	$x \geq \frac{2}{4}$
$x \leq 3$	$x \geq \frac{1}{2}$

or $\frac{1}{2} \leq x \leq 3$

graphing it



In a **disjunction** (sometimes defined as "beyond sets"), the values are limitless in two different directions, one positive, one negative. For example, in the disjunction

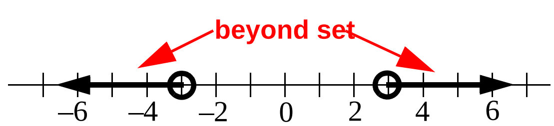
$$|x| > 3$$

the answer sets are in two opposite directions

$x > 3$ *and* $x < -3$ (greater than "+", less than "–")

$x < -3$ *or* $x > 3$

graphing it

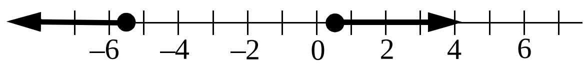


Example: $|2x + 5| \geq 6$

positive possibility	negative possibility
$2x + 5 \geq 6$	$2x + 5 \leq -6$
$2x \geq 6 - 5$	$2x \leq -6 - 5$
$2x \geq 1$	$2x \leq -11$
$x \geq \frac{1}{2}$	$x \leq -\frac{11}{2}$
$x \geq 0.5$	$x \leq -5.5$

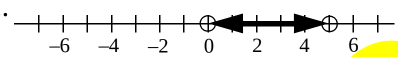
$x \leq -5.5$ *or* $x \geq 0.5$

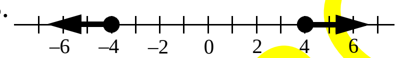
graphing it

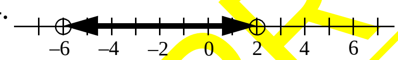


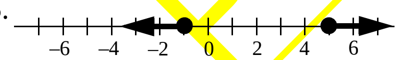
Practice:
Solve and graph 1-21. Write the—conjunction or disjunction—inequality for 22-45.

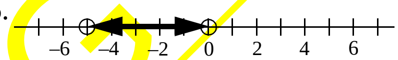
1. $|x| - 4 > 89$
2. $|y| + 4 \geq 6$
3. $|3a + 6| < 15$
4. $10 \leq 5 + |s|$
5. $13 + |x| > 19$
6. $6 < |b| + 2$
7. $|x - 17| \geq 10$
8. $|y| + 6 \leq 7$
9. $|a| + 2 > 5$
10. $26 < 13 + |s|$
11. $|2 + x| \geq 14$
12. $14 \leq |b + 6|$
13. $|x| - 15 > 5$
14. $|y + 3| < 12$
15. $20 \geq |4a + 4|$
16. $15 \leq |4 + s|$
17. $|4 + x| + 10 > 12$
18. $9 \geq |b + 2|$
19. $|x| - 2 > 6$
20. $|y + 5| < 6$
21. $|2b| + 3 \leq + 5$

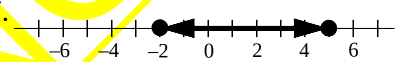
22. 

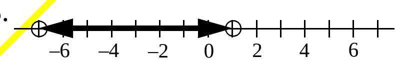
23. 

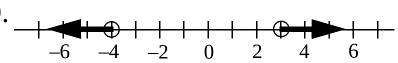
24. 

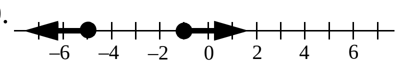
25. 

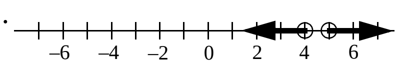
26. 

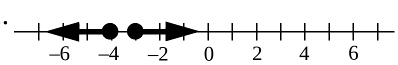
27. 

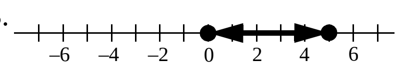
28. 

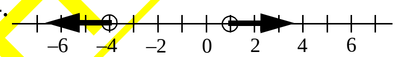
29. 

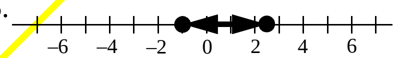
30. 

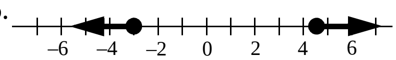
31. 

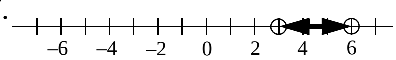
32. 

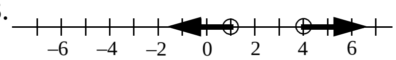
33. 

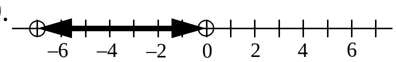
34. 

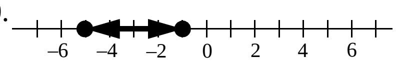
35. 

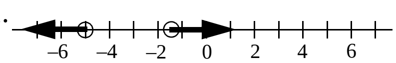
36. 

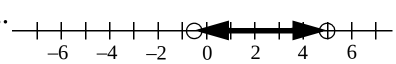
37. 

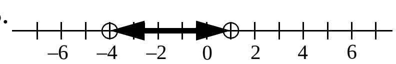
38. 

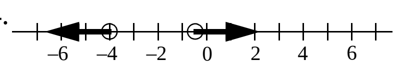
39. 

40. 

41. 

42. 

43. 

44. 

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