Section 4.1
Inequalities

Unlike equations, inequalities do not provide an exact answer to a problem. Inequalities instead tell us that the solution could be one number (any number) within a set of numbers.

For example, the equation

$$Profit = Price - Cost$$

helps us find the exact profit if the price and cost of the item is given. In plain English:

If an item costs $80, and is sold for $90, the profit is $10.

However, if we want to say "I want to make some money", then the inequality

$$Sales > Costs$$

represents a better mathematical relationship: *My costs are* $250. *Sales* > 250

Four symbols are used to show inequalities:

- $>$ to show an amount "greater than"
- $<$ to show an amount "less than"
- $\geq$ to show an amount "greater than or equal to"
- $\leq$ to show an amount "less than or equal to"

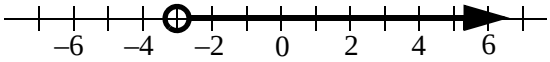
Examples:

1. $x > -3$ reads "x is greater than -3" (Answer: $-2, -1, 0, 1, 2...$)
2. $5 > y$ reads "y is less than 5" (Answer: 4, 3, 2...)
3. $a \geq 0$ reads "a is greater than or equal to 0" (Answer: 0, 1, 2, 3...)
4. $-2 \geq b$ reads "b is less than or equal to -2" (Answer: $-2, -3, -4...$)

GRAPHING INEQUALITIES

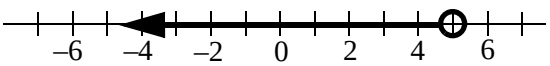
Inequalities are graphed as arrows that show the direction of the solution.

The graph of example 1 above is:

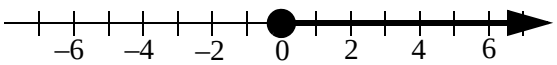


The circle around –3 means the solution does NOT reach –3.

The graph of example 2 above is:

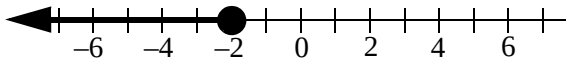


The graph of example 3 above is:



Unlike the empty circle above, a filled circle means "0" is a solution.

The graph of example 4 above is:



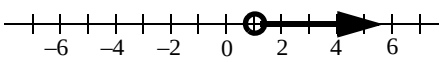
Notice the difference in plotting "greater than" and "greater than or equal." When *greater than* or *less than* are plotted, the circle marking the limit of the answer is open; when "equal" is added, the circle is filled.

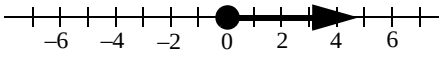
Practice:

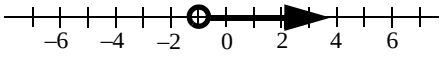
Place the sign > or < on the line and between the values to establish the correct inequality.

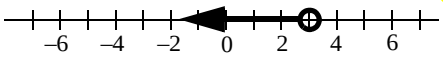
1. 4 __ 5
2. 0.45 __ –0.5
3. 0.4 __ 0.9
4. –101 __ 100
5. 6.435 __ 6.345
6. 1.2 __ –2.1
7. 0 __ 0.5
8. 9.47 __ 7.49
9. –4.6 __ –4.5
10. $\frac{7}{8}$ __ $\frac{8}{9}$
11. 0.002 __ 0.012
12. 9.5 __ 11
13. 18 __ 1.82
14. $\frac{3}{4}$ __ $\frac{4}{5}$
15. 33.3 __ 33
16. 1.2×10^2 __ 12
17. 0.008 __ –0.08
18. 9.4 __ 9.04
19. –88 __ 8.8
20. –17 __ 0.17
21. $\frac{21}{28}$ __ 0.74
22. –11 __ 1.11
23. 0.22 __ 0.205
24. 0.004 __ 0.00401

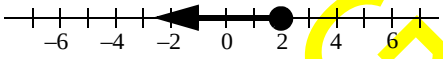
Write the inequality plotted in the line graphs.

25. 

26. 

27. 

28. 

29. 

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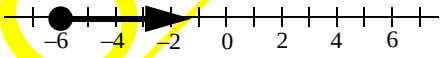
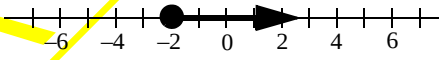
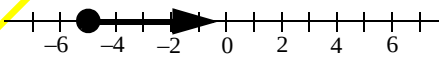
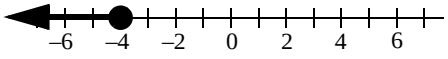
33. 

34. 

35. 

36. 

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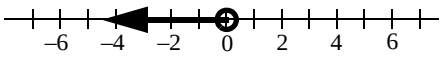
38. 

On a separate piece of paper, draw a number line and plot each inequality.

39. $x > -3$
40. $y < 5$
41. $a \geq 0$
42. $b \leq -2$
43. $8 \geq a$
44. $y < 4$
45. $-6 > x$
46. $b \leq -12$
47. $x > -1$
48. $a \geq 7$
49. $y < 2$
50. $x > -11$
51. $-7 < y$
52. $b \leq 0$
53. $x \geq -3$
54. $y < 3$
55. $1 \geq a$
56. $x > -5$
57. $b \leq 0$
58. $y < 15$
59. $x > -6$
60. $-14 \leq b$
61. $y < 5$
62. $x > -6$
63. $-1 \leq b$
64. $x > -4$
65. $x > 1.5$
66. $-8 \leq b$
67. $y < 6$
68. $10 \geq a$

ADDING AND SUBTRACTING INEQUALITIES

Inequalities are solved using the same inverse rules we use in solving equations; however, **inequalities change direction when we multiply or divide both sides of the inequality by a negative number**.

Example:

$x + 5 > 7$ — subtract 5 from both sides

$x + 5 - 5 > 7 - 5$

$x > 2$

Example:

$3x + 8 < 4x - 7$ — subtract 8 from both sides

$3x + 8 - 8 < 4x - 7 - 8$

$3x < 4x - 15$ — subtract 4x from both sides

$3x - 4x < 4x - 4x - 15$

$-x < -15$ — divide by –1 to make x positive

$\frac{-x}{-1} < \frac{-15}{-1}$

$x > 15$ — direction of inequality changed when x turned positive

MULTIPLYING AND DIVIDING INEQUALITIES

Example:

$3(x - 2) - 5x < 24$ — distribute parenthesis

$3x - 6 - 5x < 24$ — combine like terms

$-2x < 24 + 6$

$-2x < 30$ — divide by –2

$\frac{-2x}{-2} < \frac{30}{-2}$

$x > -15$ — change direction

Example:

$\frac{4x}{5} \geq \frac{8}{9}$

$\frac{5(4x)}{5} \geq \frac{8(5)}{9}$ — multiply both sides by 5 to remove 5 from left side

$4x \geq \frac{40}{9}$

$\frac{4x}{4} \geq \frac{40}{9(4)}$ — divide both sides by 4 to remove 4 from left side

$x \geq \frac{40}{36}$ reduce fraction to $x \geq \frac{10}{9}$

Example: A student's average for 9 tests is 84 points. What is the lowest score he can achieve on a tenth test to raise his average above 85 points? (Average > 85)

Make L the lowest score he could get. If the total number of points for 9 tests is

$$9 \times 84 = 756$$

Then the total number of points for 10 tests is 756 + L and the equation

$$\frac{756 + L}{10} > 85$$ defines the new average.

Solving for L

$$\begin{aligned} 756 + L &> 850 \\ L &> 850 - 756 \\ L &> 94 \end{aligned}$$

The lowest score is 95. Any score greater than 94 will raise the average above 85 points.

Practice:
Solve.

1. $4x - 14 > 26$
2. $3 + 2.5y < 9$
3. $-6y + 24 \geq -24$
4. $10 \leq 2c - 3.5$
5. $-7 - x > -10$
6. $3.5x - 6 < 1.5x + 9$
7. $20 + 10v > -16 + 4v$
8. $\frac{x}{4} + 16 \leq 5$
9. $2x - 17 \geq 15$
10. $35 + 5y > 20$
11. $-3y + 12 < -4.5$
12. $21 \leq 4c - 11$
13. $-12 - 2x \geq -20$
14. $7x - 12 > 3x + 18$
15. $12 + 5k \leq -9 + 2k$
16. $14 + \frac{a}{3} \geq 4$
17. $3x - 9 > 9$
18. $18 + 6y \geq 30$
19. $-4.5y + 54 < -18$
20. $7.5 > 2.5c - 35$
21. $-21 - 3.5x < -14$
22. $10x - 42 \leq 17x + 18$
23. $22 + 4b > -18 + 8b$
24. $3 \leq \frac{x}{7} + 13$
25. $0.3x - 3.5 < 4.5$
26. $3 + 6.5y > -13$
27. $-2.5y + 3 \geq -12$
28. $12 < c - 4$
29. $-12 - 3x > -24$
30. $1.5x - 7.5 < -2x + 8.5$
31. $4.5 + 4g \geq -6 + 5g$
32. $\frac{y}{4} + 6 \geq 12$
33. $6x - 5.5 > 11$
34. $20 + 4.5y \leq 27$
35. $-1.5y + 10 > -6$
36. $20 < 3c - 6.5$
37. $-7 - 4x \leq -10$
38. $6x - 18 > 3x + 27$
39. $31 + 4g \geq -5 - 2g$
40. $\frac{x}{4} + 10 \leq 5$
41. $5.5x - 8 > 25$
42. $-7 + 6.5y < 6$
43. $-4.5y + 10 < -34$
44. $30 \geq 14d - 6$
45. $-16 - 10x > -11$
46. $6x - 7 < 14x + 42$
47. $37 + 3u \leq -27 - u$
48. $\frac{x}{3} - 14 \geq 8$
49. $9x - 13.5 > 22.5$
50. $25 + 4y \geq 12$
51. $-2y + 20 < +19$
52. $33 \leq 24c + 38$
53. $15 - 18x > -39$
54. $4x - 21 \geq 9x + 42$
55. $13 + 3.5p \leq -8 + p$
56. $\frac{x}{13} + 14 \leq 24$
57. $10x - 17 > 33$
58. $25 + 3.5y < 14$
59. $-5y + 13 \geq -27$
60. $5 > 2c - 20$
61. $-30 - 6x < -15$
62. $-x - 1.5 > 3x + 2.5$
63. $4 + 6v \geq -9.5 + 1.5v$
64. $\frac{x}{4} - 7 \geq 2$
65. $14x - 17 > 18$
66. $-12 + 4.5y \geq -46$
67. $-4y - 7 < 12$

68. $2x - 3 < x + 4.5$
69. $6 + 4h < -21 + 3h$
70. $\frac{x}{8} + 16 \leq 2$
71. $3x - 3 > 27$
72. $-8 + 5y \geq 12$
73. $-8y + 33 < -35$
74. $11 \geq 6c - 15$
75. $-10 - 16x \leq -54$
76. $13x - 90 > 14x + 6$
77. $6 + 5r < -7 + 6r$
78. $\frac{x}{6} + 24 \geq 8$
79. $22 \leq 7c - 6$
80. $-33 - 1.5x \geq -112$
81. $4.5x - 27 \leq 2.5x + 34$
82. $110 + 5v < -7 + 2v$
83. $4x - 17 \leq 34$
84. $23x - 36 > 18$
85. $70 + 7y \geq 21$
86. $-2y + 1.6 < -2.3$
87. $3.6 \leq 0.6c - 2.4$
88. $-155 - 21x \geq -240$
89. $6x - 1.2 < 2x + 1.4$
90. $45 + 4k > -90 + 6k$
91. $-7 + \frac{a}{9} \leq 114$
92. $4x - 1.4 \geq 2.8$
93. $144 + 9y \leq 225$
94. $-3.1y + 6.2 > -3.4$
95. $13.8 < c - 2.24$
96. $-6.4 - 7x \geq -2.8$
97. $0.4x - 7.3 > 0.8x + 7$
98. $4.1 + 8b \leq -7 + 5b$
99. $12.9 \geq \frac{x}{3} + 14$
100. $0.2x - 0.9 < 2.8$
101. $12 + 3y > -111$
102. $-16y + 80 \geq -125$
103. $9 \leq 5c - 77$
104. $-13 - 7x > -23$
105. $18x - 135 < -22x + 556$
106. $48 + 8f \leq -22 + 28f$
107. $\frac{y}{7} + 20 \leq 19$
108. $15x - 15 \geq 25$
109. $26x + 13 > 39$
110. $-9y + 45 < -135$

Write the sentence as an inequality.

1. 7 is less than an amount x.
2. Half an amount is more than or equal to 27.
3. 8.5 is less than the amount p.
4. An amount greater than or equal to 100.
5. An amount that is at least 12.
6. 45 more than a number is greater than 20.
7. 8 is greater than an amount a.
8. One fifth of an amount is less than or equal to 30.
9. 2.8 is more than amount n.
10. An amount less than 55.
11. An amount less than or equal to 66.
12. 20 more than a number is less than 5.
13. 5 more than one half of an amount is less than 7.
14. 16 is less than or equal to 5 less than an amount.
15. 6 more than twice an amount is greater than another amount.
16. An amount is greater than half of another amount.

Solve.

17. Ross's average bowling 11 games is 179 points. What is the lowest score he must achieve in the 12th game to maintain his average to at least 170 points?
18. The sum of three consecutive integers is less than 63. What are the largest values of the three numbers?
19. The perimeter of a rectangle is at least 72 feet. If the length is twice the width, find the smallest integers that could be used to form a rectangle.
20. The sum of three consecutive even integers is greater than 120. What are the lowest possible values of the integers?
21. The batting average of a baseball player is 250 (0.25). If he has been to the plate 140 times, how many consecutive hits does he need to reach an average of at least 275 (0.275)?