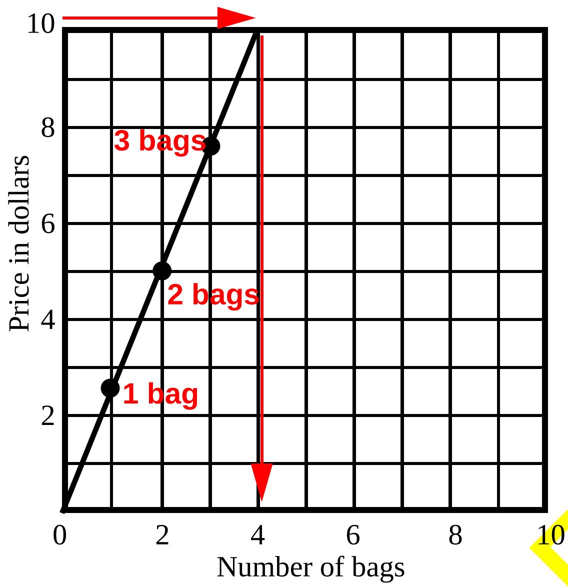
Section 3.6
Linear Models

By now you should have a basic idea of what "linear" in mathematics means: A way of connecting the relationship between two variables, where the relationship in question may be represented by a straight line. Therefore, the "linear" equations we have been discussing in the previous sections of chapter three, help us solve "linear problems."

At the beginning of section 3.2, the graph shown below is given as an example representing a relationship between two variables, in this case bags and price.



What is a model?

In mathematics, a model is a reproduction or replica that is capable of projecting on what we already know. In other words, mathematical models use historical data to predict different make-believe outcomes to help us make decisions. In this section, only linear models will be discussed.

An equation describes the situation.

Because it helps us in predicting how much any number of bags would cost, the graph to the left is a linear model. Using what we already know, the slope of the line is:

$m = \frac{5}{2}$ and the y-intercept is zero

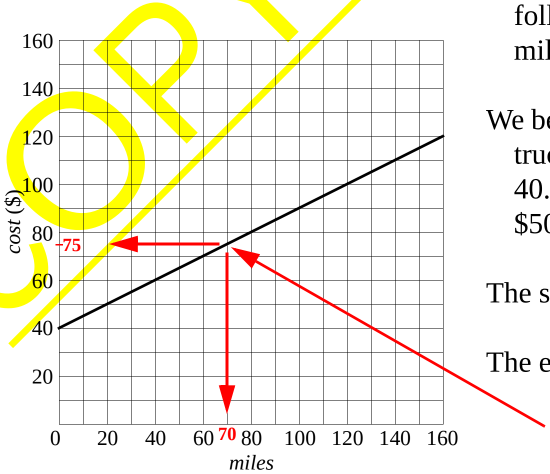
$y = mx + b$ $b = 0$

The equation of the line is $y = \frac{5}{2}x$

We can now compute the price of any number of bags by the use of the **linear model** represented by the equation. X represents the number of bags and y the price. Or we can change the variable x for b (bags) and y for p (price) to make it $p = \frac{5}{2}b$. For example, if b is 150 then p is: $p = \frac{5 \times 150}{2} = \$375$

Example:

To move, Pat needs to rent a truck for a few hours. Build a linear model to predict her costs using the following specifications: It costs \$40 per day and \$0.50 per mile to rent the truck she wants.



We begin with the condition that Pat must pay \$40 for starting the truck (zero miles). Therefore, the line for this model starts at 40. From 40 the line climbs at the rate of 50 cents per mile or \$50 per hundred miles. The graph shows this.

The slope, m, is 0.5 (50/100) and the y-intercept is 40 ($miles = 0$)

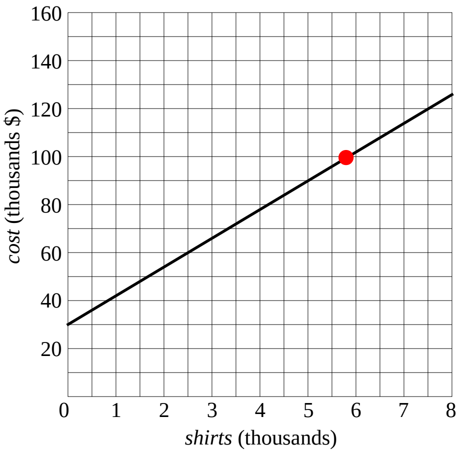
(m = *miles*)

The equation is $Cost = 0.5m + 40$ or $y = 0.5x + 40$

If she drives 70 miles: $y = 0.5(70) + 40 = \$75$

Example:

Ernestine wants to start a business making shirts. The machinery, and other equipment to do the manufacturing of the shirts, costs $30,000. She computed the material and labor costs of making one shirt and she arrived at a figure of $12. Write the equation that would describe Ernestine's linear model.



Before the first shirt is made, she pays $30,000. Thus the first shirt will cost $30,012, the second shirt $30,024, an so on. Thus the slope of the line is the vertical change ($y = 12$) over the horizontal change ($x = 1$):

$m = \frac{12}{1} = 12$ and the y-intercept (when $x = 0$) is 30,000

Ernestine's linear model is: $y = 12x + 30{,}000$

5,580 shirts will cost (red dot in graph) $y = 12(5580) + 30000 = \$96{,}960$

Practice:

1. A recipe calls for five minutes of cooking time for every three pounds of meat. Write an equation that defines this linear model and then find how long it would take to cook 37 pounds of meat.
2. The weight of an empty truck is 12,000 pounds. If it carries boxes that weigh 32 pounds each, write a linear equation for this model and calculate the total weight when it carries 92 boxes.
3. A contract requires renters to pay $2,000 for the use of a dance hall, plus $2 for every dancer that participates. Write an equation that shows this and find the rental cost for 122 dancers.
4. Brian gets paid a flat rate of $50 per day to teach math, plus $10 for every student. If he has 28 students, write the linear equation that shows this and calculate his pay.
5. Harold gets five cents for every spray can he sells and a $300 bonus when he sells more than 5000 cans. Write the linear equation for his compensation model and compute his pay for 6570 spray cans.
6. Beets are sold at twelve dollars per five pounds. Write a linear equation that describes this model and find the cost of 2.25 pounds of beets.
7. To plant mangoes, farmer Jerry needs $12,000 to prepare the land and $18 per seedling. Write a linear equation that shows this relationship and find Jerry's cost for planting 211 trees.
8. An airplane that weighs 70 tons carries 0.25-ton containers. Write the airplane's weight linear model and then find how heavy is the plane when it takes off with 53 containers.
9. While planning the school's prom, Stephanie calculates that the dance hall rents for $1,500 per night, and that each meal will cost an additional $31 per person. Write the equation that represents the "prom night" model and find the cost of inviting 125 students and family.
10. A manufacturer's contract states that a salesperson will be paid a base salary of $700 per week plus $11 for every unit sold. Write an equation for this linear model and then find a week's salary for 103 units.
11. For every floral design Flora makes, she gets $19; however, if no floral designs are ordered, she still gets her $50 per day base pay. Write the linear equation of her model and her take for 25 designs.
12. To move, Nat rents a truck for a few hours. Build a linear model to predict his costs using the following specifications: It costs $52 per day and $0.38 per mile to rent the truck he wants.