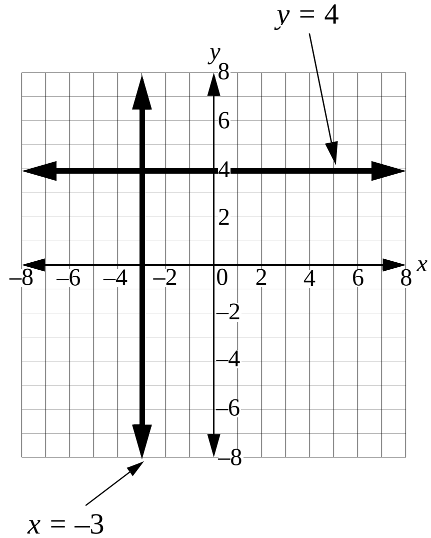
Section 3.5
From Equation to Line, from Line to Equation

$y = 4$ (1)

$x = -3$ (2)

$y = -2x + 1$ (3)

Equation 1 on the right represents a horizontal line. It is a horizontal line because the slope (the coefficient of *x*) is zero. We know it is zero because to plot a line we always start with the slope-intercept equation $y = mx + b$ and the "*mx*" portion of the equation gets eliminated when $m = 0$, leaving only the value of *b* (the *y*-intercept) in the equation [$y = (0)m + 4$].



If $y = (0)x + 4$ then $y = 4$

Equation 2 above is a vertical line. For vertical lines, the slope is undefined (infinite) because the *change* of *x* is zero. NOTICE EQUATIONS HAVING ONLY ONE VARIABLE ARE EITHER HORIZONTAL OR VERTICAL.

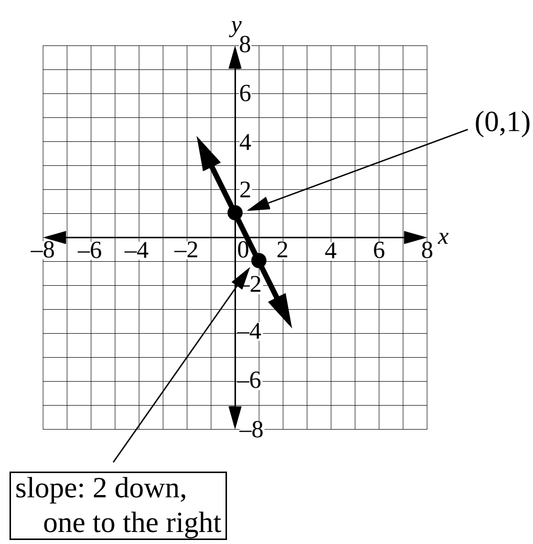
Equation 3 represents a diagonal line in the *slope-intercept* form.

Plotting a line using the slope-intercept form

The *slope-intercept* form, as its name indicates, provides us with the *y*-intercept and *slope* values by just looking at it.

$$y = mx + b$$

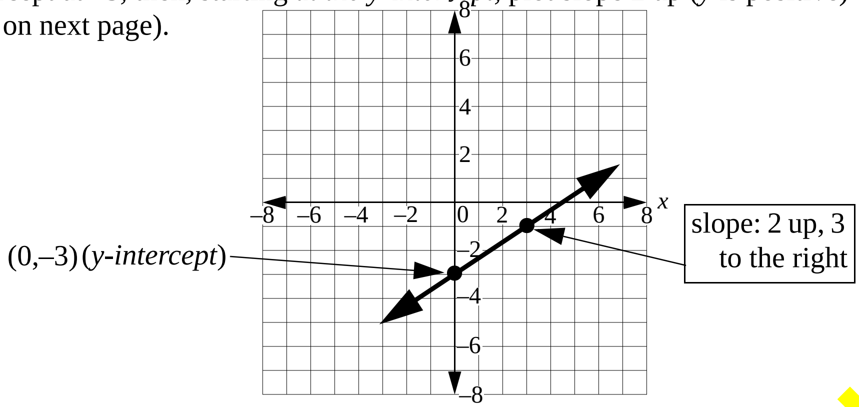
Where *m* is the slope and *b* is the *y*-intercept. For equation 3 above, the *y*-intercept value is 1 and the *slope* is –2.



We plot the *y-intercept* (the place where the line will cross the *y-axis*) FIRST—in this case (0,1) because, for the *y*-intercept, the value of *x* is always zero. Secondly, we find the next point by moving two spaces down and one to the right because the slope is $m = \dfrac{change\ in\ y}{change\ in\ x}$ or $\dfrac{-2}{1}$ and *y*, being negative, moves down and *x*, being positive, moves to the right. Only two points are needed to form a line. Join both points and you have drawn the line that defines the equation. See graph to the right.

Example: Plot equation $y = \frac{2}{3}x - 3$

First plot y-intercept at –3, then, starting *at the y-intercept*, plot slope 2 up (y is positive) and 3 to the right (x). (See plot on next page).



Example: Plot equation $2x - 5y = 10$

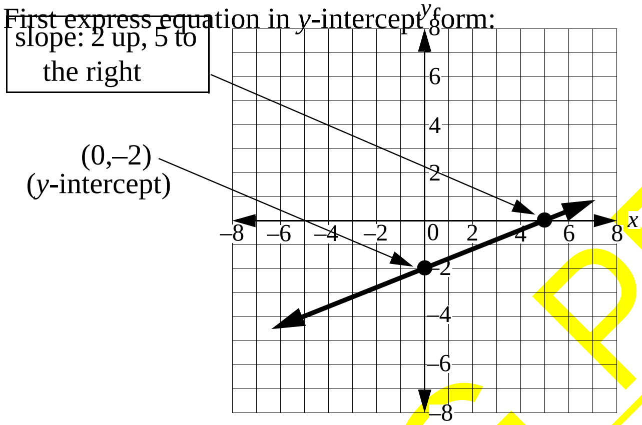
First express equation in y-intercept form:

$$-5y = -2x + 10$$

$$\frac{-5}{-5}y = -\frac{2}{-5}x + \frac{10}{-5}$$

$$y = \frac{2}{5}x - 2$$

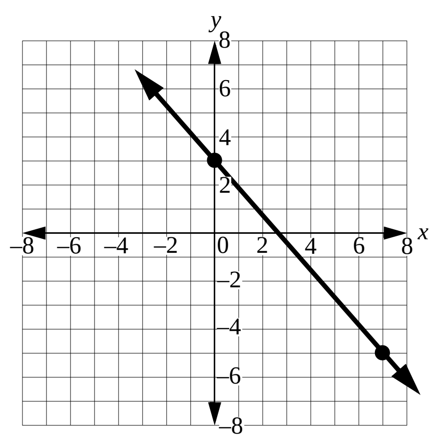
y-intercept = –2 $\qquad$ slope = $\frac{2}{5}$



From a graph, find the equation of the line.

In the graph to the right, the line crosses the *y-axis* at 3 and the *slope* (the y change between the two points is negative—8 down—over the x change which is 7) is $-\frac{8}{7}$.

Using these numbers in the slope-intercept form ($y = mx + b$), $m = -\frac{8}{7}$ and $b = 3$. Therefore, $y = -\frac{8}{7}x + 3$.



Practice:

On a sheet of graph paper, plot the following equations.

1. $y = x + 2$
2. $y = -2x + 3$
3. $y = 3x - 5$
4. $x + \frac{1}{2}y = 8$
5. $2x - 3y = 9$
6. $y - 4x = 1$
7. $8 = 3x - 4y$
8. $5y = 2x - 10$
9. $\frac{1}{2}x + y = 3$
10. $12 = 2y - 3x$
11. $x + y = 4$
12. $y = \frac{3}{4}x - 4$
13. $9 = \frac{1}{4}x + y$
14. $5x - 2y = 8$
15. $y = 5$
16. $4y + x = 8$
17. $3y - x = 12$
18. $x = -3$
19. $4y + 5x = 20$
20. $y = -3x + 1$
21. $6x - y = 5$
22. $9 = 3y + 2x$
23. $3x = 15 + 5y$

Write the equation for the lines shown in the graphs below.

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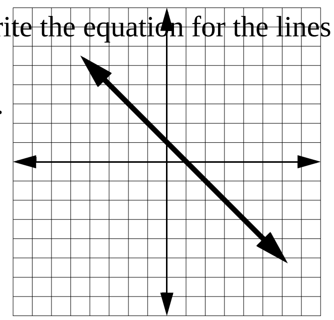

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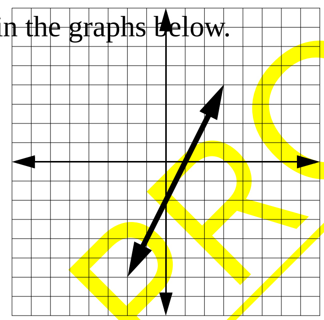

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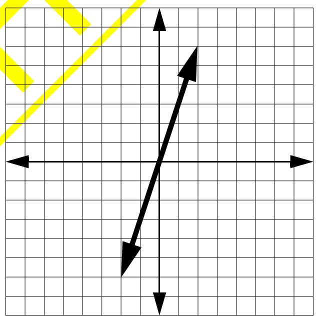

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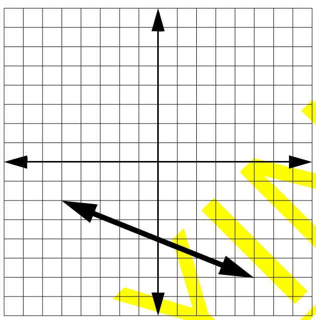

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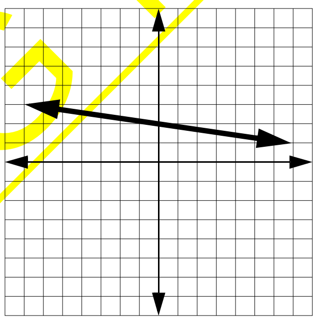

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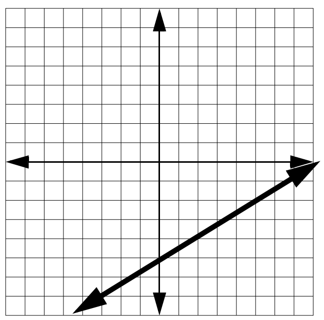

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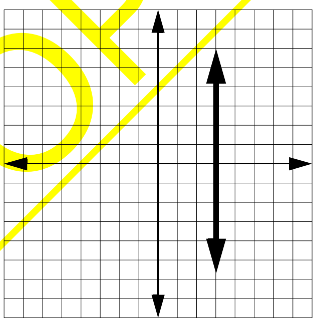

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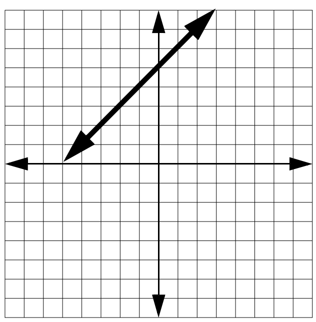

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