Section 3.4
Parallel and Intersecting Slopes

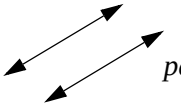

Figure 1

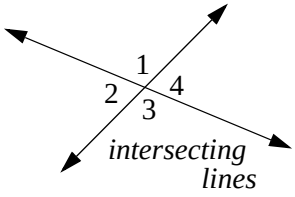

Figure 2

The multiple lines we draw on a plane (or a piece of paper) may cross or not cross at all. If they do NOT cross, lines are said to be parallel; if they cross, they will form four angles.

When the four angles are equal, they measure 90º each. Lines that cross at 90º are said to be perpendicular (⊥).

The way to tell if two equations represent lines that are parallel or perpendicular is by the slope: If the slopes of two or more lines are equal, the lines must be parallel (Figure 1) and the distance between any two lines is always the same. If, on the other hand, the slopes are not equal, it means the lines cross (Figure 2). If the lines cross at 90º, then the slopes are *reciprocal* AND *opposites* of each other (Figure 3).

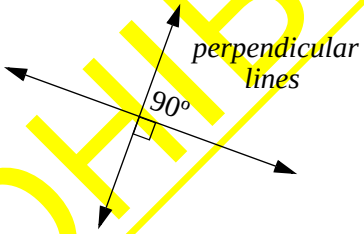

Figure 3

A *reciprocal* is a rational number that is written upside-down:

the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ the reciprocal of $\frac{3}{1}$ is $\frac{1}{3}$ the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2

The *opposite* of a number is the number with the "sign" changed:

the opposite of 4 is –4 the opposite of $-\frac{1}{5}$ is $\frac{1}{5}$

Therefore,

the *reciprocal* AND *opposite* of $\frac{1}{4}$ is $-\frac{4}{1}$ or –4

the reciprocal opposite of $-\frac{5}{4}$ is $\frac{4}{5}$

Because $\frac{1}{1} = 1$, the reciprocal opposite of 1 is –1

Example:
Are lines represented by equations $y = 2x - 7$ and $x + 2y = 5$
parallel, perpendicular, or neither of the two?

By looking at the first equation, we can identify the slope as the coefficient of x. Therefore, the slope of the first equation, $y = 2x - 7$, is 2.

Now we write the second equation in *y*-intercept form by subtracting *x* from both sides.

$$\begin{aligned} x + 2y &= 5 \\ x - x + 2y &= 5 - x \\ 2y &= -x + 5 \end{aligned}$$

Dividing both sides by 2 $\quad \frac{2}{2}y = -\frac{x}{2} + \frac{5}{2}$

yields the *y-intercept* equation. $\quad y = -\frac{1}{2}x + \frac{5}{2}$

The slope of the second line is $-\frac{1}{2}$ (coefficient of *x*).

Because the slopes, 2 and $-\frac{1}{2}$, are *reciprocal* AND *opposites*, the lines are perpendicular.

Example:
Are the lines represented by equations $3x - 2y = 14$ and $2x - 3y = -15$ parallel, perpendicular, or neither of the two?

Turn first equation into *y*-intercept.

$$\begin{aligned} 3x - 2y &= 14 \\ 3x - 3x - 2y &= -3x + 14 \\ -2y &= -3x + 14 \\ y &= \frac{-3}{-2}x - \frac{14}{2} \\ y &= \frac{3}{2}x - 7 \end{aligned}$$

The slope is $\frac{3}{2}$ (coefficient of *x*).

Turn second equation into *y-intercept*.

$$\begin{aligned} 2x - 3y &= -15 \\ 2x - 2x - 3y &= -2x - 15 \\ -3y &= -2x - 15 \\ y &= \frac{-2}{-3}x - \frac{15}{-3} \\ y &= \frac{2}{3}x + 5 \end{aligned}$$

The slope is $\frac{2}{3}$ (coefficient of *x*).

Because the slopes, $\frac{3}{2}$ and $\frac{2}{3}$, are neither *equal* nor *reciprocal opposites*, the lines cross, but are not perpendicular.

Example:
Are the lines represented by equations $y = x - 6$ and $3x - 3y = 11$ parallel, perpendicular, or neither of the two?

The slope of the first equation, taken from the coefficient of *x*, is 1.

Turn second equation into *y*-intercept form.

$$\begin{aligned} 3x - 3y &= 11 \\ 3x - 3x - 3y &= -3x + 11 \\ -3y &= -3x + 11 \\ y &= -\frac{3}{-3}x + \frac{11}{-3} \\ y &= x - \frac{11}{3} \end{aligned}$$

The slope is 1 (coefficient of *x*). Because the slopes, 1 and 1, are equal, the lines are parallel.

Practice:
Find the slope and determine if the lines are parallel, perpendicular, or neither.

1. $3y = 5x + 21$
   $-5x = -3y - 6$

2. $x + 4 = y$
   $y - x = -3$

3. $\frac{2}{3}x + y = 6$
   $8y - 12x - 12 = 0$

4. $6y - x = -12$
   $\frac{1}{6}x + y = 3$

5. $5y - 10x = 3$
   $8x - 4y = 16$

6. $6x + 3y = 4$
   $-4 = y + 2x$

7. $2 = -y + \frac{3}{2}x$
   $\frac{2}{3}x + y - 9 = 0$

8. $-8 = -y + \frac{1}{3}x$
   $-\frac{3}{2}x + y + 4 = 0$

9. $3x - y = -6$
   $-3x + y = 4$

10. $-2x + y = 5$
    $y - 6 = -6x$

11. $3x - y = -9$
    $2y - 6x = -2$

12. $4x + y = 2$
    $-\frac{1}{4}x = -y - 4$

13. $-7x + y = -8$
    $2 = -y + 7x$

14. $9y + 3x = 2$
    $4y = x + 5$

15. $6x - y = 4$
    $6x + y = -4$

16. $12y = 4x + 20$
    $2x + 6y = -3$

17. $y = -7x - 5$
    $2y = -7x - 10$

18. $y = 2x + 7$
    $5y + 10x = 20$

19. $3x = -y + 2$
    $y = -8 - 3x$

20. $x - y = -1$
    $y = -x + 8$

21. $-5 = -2y + 8x$
    $y = -4x + 2$

22. $5x + 2y = 6$
    $2x - 5y = -3$

23. $-1 = -y + \frac{2}{3}x$
    $\frac{3}{2}x + y = 5$

24. Write an equation for the line which is parallel to $3x - 2y = 14$ and passes through point $(-2,4)$.

25. Write an equation for the line which is perpendicular to $y - x = 3$ and passes through point $(-3,0)$.

26. Write an equation for the line which is parallel to $2y = 5x - 10$ and passes through point $(4,1)$.

27. Write an equation for the line which is parallel to $x - y = 1$ and passes through point $(4,-3)$.

28. Write an equation for the line which is perpendicular to $4 = -2y + x$ and passes through point $(-7,2)$.

29. Write an equation for the line which is perpendicular to $y = 3x - 7$ and passes through point $(0,0)$.

30. Write an equation for the line which is perpendicular to $5x - 2y = 12$ and passes through point $(1,-1)$.

31. Write an equation for the line which is parallel to $0 = y - 6x$ and passes through point $(8,-5)$.