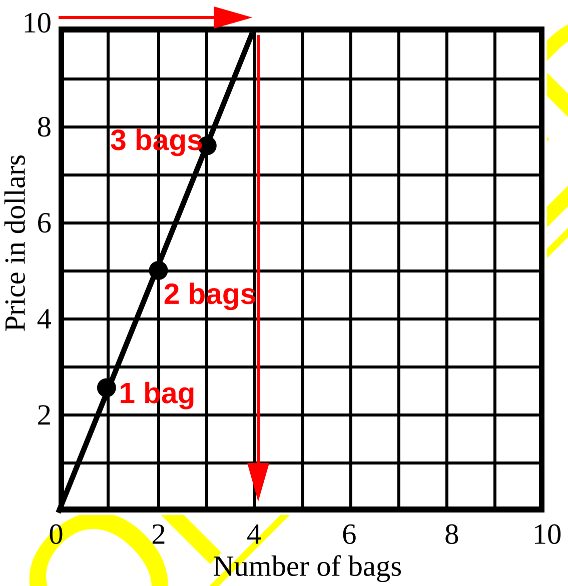
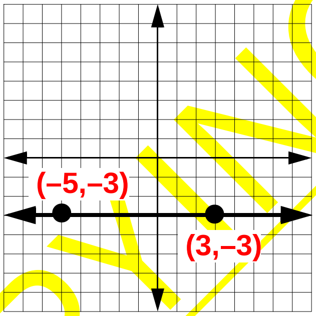
Section 3.2
From Linear Pattern to Equation

A person goes to the supermarket and reads that the price of a bag of sugar is \$2.50. This means the price of two bags is \$5.00 (2 times 2.50) and three bags \$7.50 (3 times 2.50). Or perhaps he reads that three apples go for \$0.25, and six apples, \$0.50, and so on. These examples are what is called a "linear pattern." Linear patterns, when plotted, form a line. The "bag of sugar" linear example (see graph) can be plotted to show this.

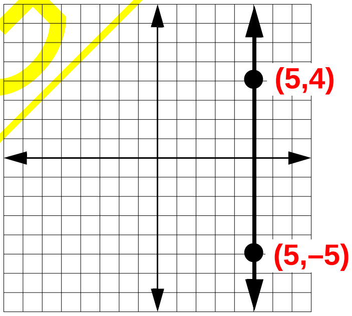


Necessary information to plot a line:
A line is made up of points and at least two points are needed to plot a line. Because lines must be drawn on a surface (two dimensions), each point's location needs two numbers. Called coordinates, these numbers are represented by letters (*x* and *y* are the most commonly used). Therefore, to plot a line, four numbers are needed: an *x* and a *y* for point one, and an *x* and a *y* for point two. The example, the graph matches 1 bag (*x* value) and \$2.50 (*y* value), 2 bags and \$5, 3 and 7.5. How many bags will you get for \$10? Answer: four bags.

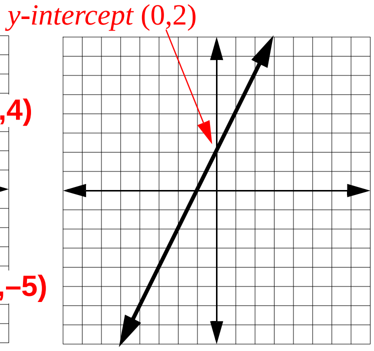
How to turn a line into an equation:
Straight lines on a flat surface can be drawn *horizontally*, *vertically* and *diagonally*. When plotted on a graph, the *y* values of all points on a *horizontal* line are always the same (left graph); conversely, the *x* values of all points on a *vertical* line are always the same (center graph). Each point of a *diagonal* line has different values for *x* and *y* (right graph). To define—or locate—a straight line on a flat surface, it is best to show where the line crosses the vertical axis (*y* axis) and what the *slope* of the line is. The *y-intercept* (right graph) tells us where to find the first point of the line, and the slope shows the direction the line takes after crossing the *y* axis.



horizontal, $y = -3$

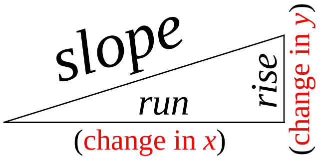


vertical, $x = 4$



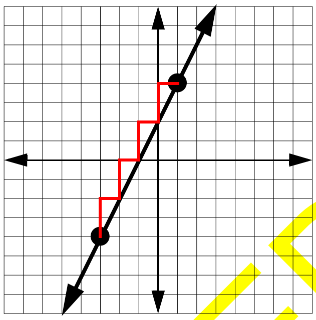
diagonal, $y = 2x + 2$

How to find the slope: A slope is like a ramp, where the rate of climb is determined by comparing the vertical distance (*rise*) over the hor-



$$slope = \frac{change\ in\ y}{change\ in\ x}$$

izontal distance (*run*). In the case of a graph, it means that the slope is a ratio (a fraction) of the change of y (vertical) over a particular change of x (horizontal).



For example, the diagonal line shown to the right goes through points (–3, –4) and (1, 4) and has a slope of 2. The slope is 2 because for every time y climbs (changes) two units, x moves to the right once $\left[\frac{y}{x} = \frac{2}{1}\right]$. At the same time, the y-intercept is where the line crosses the y-axis and in this particular case is 2. Therefore, the equation is written as:

$$y = 2x + 2$$
$$y = \underset{\text{slope}}{m}x + \underset{y\text{-intercept}}{b}$$

where m, the slope, is always the coefficient (number in front) of x, and b is the y-intercept (the point where the line crosses the y axis).

Example:
Write the equation for the line whose slope is 3 and y-intercept 2.

Slope = 3 = m y-intercept = 2 = b Answer: $y = 3x + 2$

Example:

Write the equation for the line whose slope is $-\frac{3}{2}$ and y-intercept 5.

$m = -\frac{3}{2}$ y-intercept = 5 Answer: $y = -\frac{3}{2}x + 5$

Example:
Write the equation for the line whose slope is –1 and y-intercept –3.

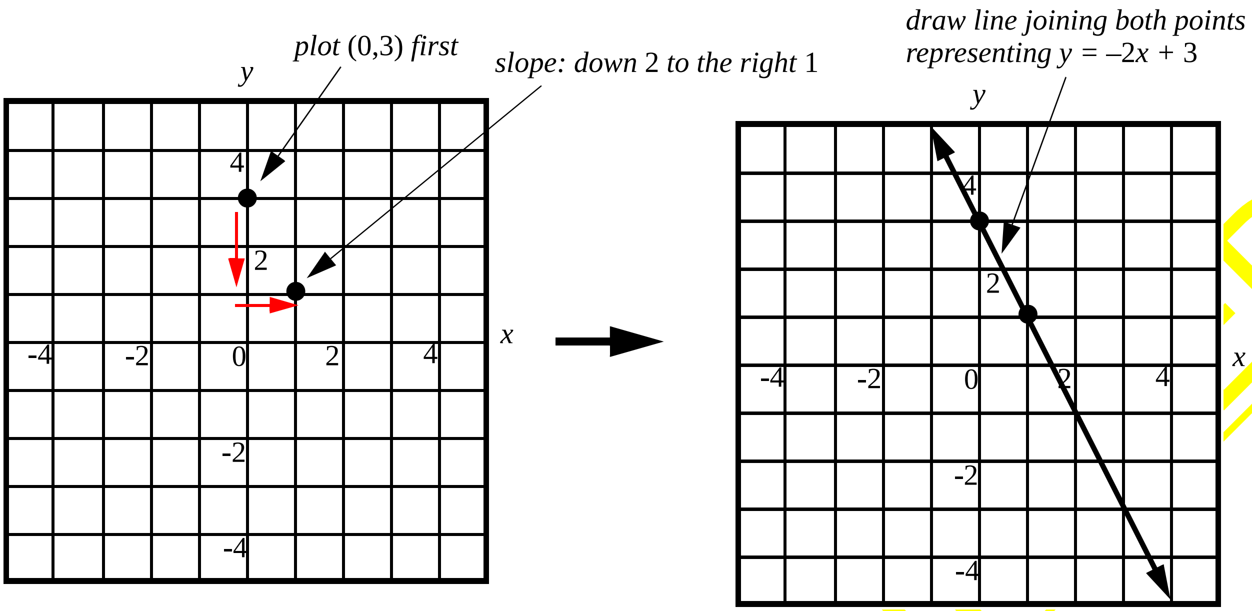
$m = -1$ y-intercept = –3 Answer: $y = -x - 3$

DOING IT BACKWARDS (plotting the line of an equation)

Example:
On a graph, plot line $y = -2x + 3$ (see top two graphs next page)

First plot the y-intercept value +3. Because the slope, $-\frac{2}{1}$, is the ratio of the *change in y* (–2) over *the change in x* (1), then this slope means that whenever the value of y moves down (negative) twice, the value of x will move once to the right (The slope is negative when only one of the variables is negative.). So the second point of the line is found by moving down to point (0,1) on the y axis (two spaces) and then one to the right to point (1,1). Finally, draw the line representing

$$y = -2x + 3.$$



plot (0,3) *first*

slope: down 2 *to the right* 1

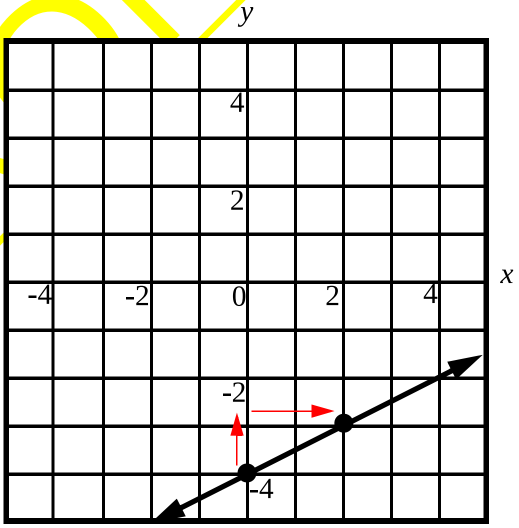
draw line joining both points representing $y = -2x + 3$

Example: Plot line $y = \frac{1}{2}x - 4$

Plot *y*-intercept value at –4.

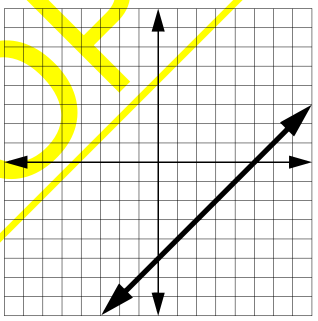
Second point: Using the slope, $\frac{1}{2}$, UP one (*y*), to the RIGHT 2 (*x*). Draw line that passes through the two points.

Remember:
When the slope is positive, move up (+*y*) and to the right (+*x*), and when is negative move down (–*y*) and to the right (+*x*). The *x* (horizontal) move is always to the right.



Practice
Write the linear equation that represents each of the lines shown.

1.



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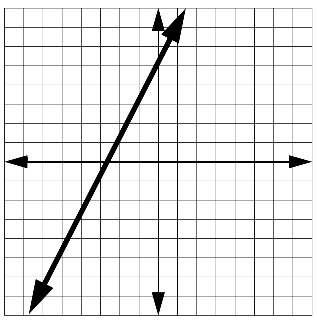

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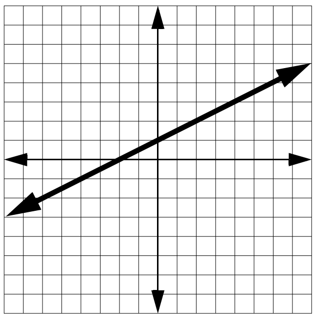

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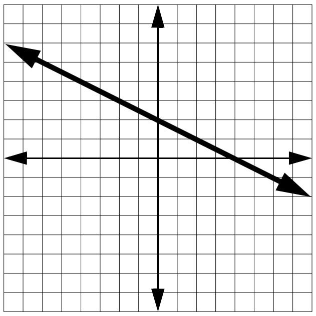

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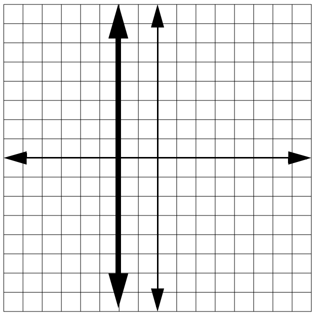

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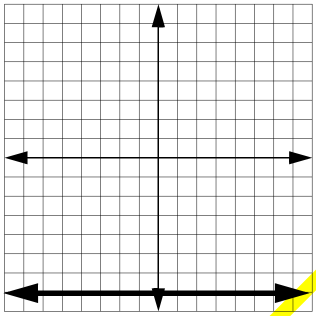

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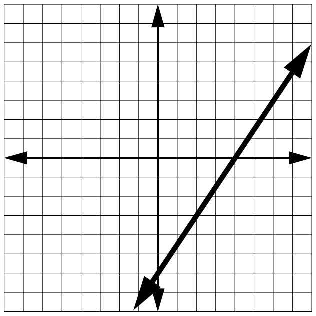

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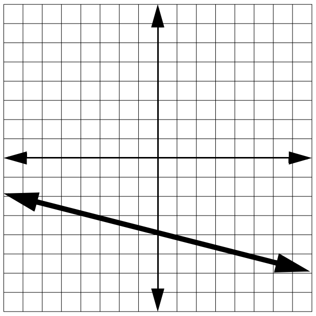

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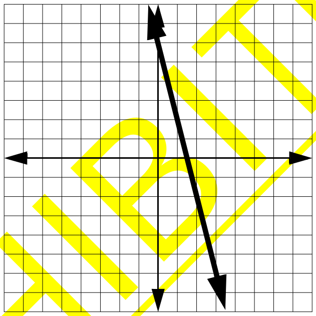

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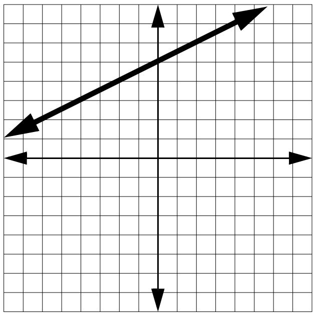

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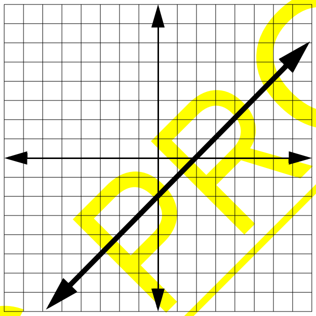

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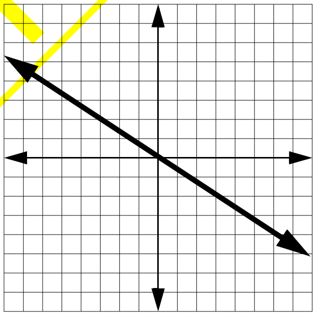

On separate graph paper, plot the line that represents each of the following equations.

13. $y = x + 3$
14. $y = x - 2$
15. $y = 2x - 1$
16. $y = -x + 5$
17. $y = -3x - 4$
18. $y = \frac{1}{2}x + 7$
19. $y = -2x - 8$
20. $y = \frac{3}{2}x + 13$
21. $y = -\frac{8}{5}x + \frac{3}{8}$
22. $y = 2.5x - 10$
23. $y = 4.2x - 5$
24. $y = \frac{3}{4}x + \frac{3}{2}$
25. $y = \frac{2}{3}x + 8$
26. $y = 1.5x - \frac{3}{4}$
27. $y = -\frac{5}{2}x + 2.5$
28. $y = \frac{1}{2}x - 1.4$
29. $y = 7x - 9$
30. $y = 3.5x + \frac{1}{4}$
31. $y = \frac{4}{3}x - 5.5$
32. $y = 1.8x - 3$
33. $y = -\frac{7}{2}x + 1.5$
34. $y = 3.5x + \frac{5}{6}$
35. $y = 5x + 4$
36. $y = \frac{2}{5}x - 3.8$
37. $y = 6.5x - 2.6$
38. $y = 2.4x + 1$
39. $y = -\frac{1}{2}x + \frac{1}{6}$
40. $y = \frac{1}{8}x - 6.5$
41. $y = -\frac{9}{2}x - 7$