

Section 2.1

Equations: Addition and Subtraction

An equation is an open statement that has one or more unknowns, also called variables.

Example:

$$x + 5 = 12 \quad \text{or} \quad y - 15 = 8 \quad (x \text{ and } y \text{ are variables})$$

Equations are solved by the use of *inverse* rules (see page 16) and doing the opposite. The value of the unknown is found because inverse rules undo what the equation states. A typical inverse rule says additions are undone by subtractions and subtractions with additions. Therefore, the first equation above is:

$$\begin{aligned} x + 5 - 5 &= 12 - 5 && \text{(addition undone by subtraction)} \\ x + 0 &= 7 \\ x &= 7 \end{aligned}$$

Notice that by undoing the +5 on the left with a -5, we are forced to add -5 on the right side of the equation to keep the equation balanced (equal). The value of x is then exposed as $12 - 5 = 7$.

In the second equation above, -15 is being subtracted from an unknown "y."

$$\begin{aligned} y - 15 + 15 &= 8 + 15 && \text{(subtraction undone by addition)} \\ y - 0 &= 8 + 15 \\ y &= 23 \end{aligned}$$

Notice that by undoing -15 on the left with a +15, we are forced to add +15 on the right side of the equation to keep the equation balanced (equal). The value of y is then exposed as $8 + 15 = 23$.

Example:

$$\begin{aligned} x - 6 &= 10 \\ x - 6 + 6 &= 10 + 6 \\ x + 0 &= 16 \\ x &= 16 \end{aligned}$$

Example:

$$\begin{aligned} 2x + 3 &= x - 12 \\ 2x - x + 3 - 3 &= x - x - 12 - 3 \\ x + 0 &= 0 - 12 - 3 \\ x &= -15 \end{aligned}$$

Example:

$$\begin{aligned} 13 &= a + 10 \\ 13 - 10 &= a + 10 - 10 \\ 3 &= a + 0 \\ 3 &= a \\ a &= 3 \end{aligned}$$

"Sign sign everywhere a sign.... do this, don't do that, can't you read the sign?"*

Understanding how to handle positive and negative signs is hard for students entering algebra for the first time. Hopefully, the following examples and brief explanation will help.

$$8 + 10 - 3 + -5 - + 7 - - 6 + (-12) - (14) - (-9) = -8$$

In this example, 8 is added to 10 and then 3 is subtracted, -5 is added, 7 is subtracted, and -6 is subtracted... This shows we can *add negative* numbers (+ -5), *subtract positive* numbers (- + 7), *subtract negative* numbers (- - 6), and so on. Sounds confusing? Let's break it down using the number line (see page 21):

1. Start by adding 8 and 10 to get 18, and taking 3 away is 15. (So far, so good!).
 2. Next, "add negative 5" (+ -5) to 15 and go down to 10 (adding "bad" to "good" makes it go down).
 3. Next, "subtract positive 7" (- + 7) and go down to 3 this time (taking "good" away makes it go down).
 4. Next, "subtract negative 6" (- - 6) and go up to 9 (taking "bad" away makes it "good", and goes up).
 5. At this point, having a parenthesis is the same as not having one. Ignore it and continue.
 6. Next, "add negative 12" [+ (-12)] and go down beyond zero to -3 (the negative side of the number line).
 7. Next, "subtract positive 14" [- (14)] and go down to -17 (taking "good" away makes it go down further).
 8. To finish, "subtract negative 9" [- (-9)] and go up to -8 (taking "bad" away makes it go up).
- (Notice the similarities between step 6 and step 2, step 7 and step 3 and step 8 and step 4.)

Taking a short cut

$$-12 + 9 - (-14) - (7) + (-11) - (+8) + 5 - -16 + - 2 = 4$$

Continues in page 30.

Practice:

Solve.

1. $x - 5 = 8$
2. $y + 10 = 17$
3. $2a + 3 = a + 7$
4. $21 = 14 + s$
5. $34 - x = 19$
6. $7 = -b + 18$
7. $x - 7 = 18$
8. $y + 6 = 27$
9. $3a + 8 = 2a + 56$
10. $6 = 13 + s$
11. $12 - x = 45$
12. $4 = -b + 36$
13. $x - 15 = 54$
14. $y + 5 = 8$
15. $5a + 23 = 4a + 83$
16. $7 = 24 + s$
17. $42 - x = 27$
18. $9 = -b + 24$
19. $x - 23 = 66$
20. $y + 23 = 64$
21. $2b + 13 = b + 5$
22. $-28 = 33 + t$
23. $17 - y = 29$
24. $25 = 13 - b$
25. $y - 13 = 72$
26. $u + 12 = 62$
27. $2c + 31 = c + 6$
28. $9 = 21 + p$
29. $43 - g = 92$
30. $16 = -v + 4$
31. $f - 15 = 14$
32. $h + 6 = 12$
33. $2t + 61 = t + 8$
34. $21 = 14 + s$
35. $5 - x = 17$
36. $13 = -c + 78$
37. $x - 30 = 45$
38. $y + 8 = 13$
39. $2a + 8 = a + 10$
40. $12 = 41 + s$
41. $17 - x = 7$
42. $14 = -b + 9$
43. $x - 9 = 23$
44. $y + 12 = 34$
45. $3a + 4 = 2a + 6$
46. $56 = 7 + s$
47. $23 - x = 45$
48. $43 = -b + 24$
49. $x - 15 = 32$
50. $y + 67 = 54$
51. $8a + 33 = 7a + 87$
52. $67 = 24 + s$
53. $19 - x = 54$
54. $3 = -b + 65$
55. $x - 18 = 28$
56. $y + 17 = 83$
57. $2y + 71 = y + 23$
58. $51 = 49 + t$
59. $71 - d = 72$
60. $95 = -g + 42$
61. $x - 55 = 77$
62. $r + 72 = 28$
63. $2d + 6 = d + 22$
64. $43 = 54 + u$
65. $43 - s = 66$
66. $32 = -p + 65$
67. $x - 51 = 76$
68. $u + 87 = 95$
69. $2h + 55 = h + 5$
70. $31 = 65 + s$
71. $32 - x = 84$
72. $15 = -b + 18$
73. $x - 25 = 66$
74. $y + 88 = 46$
75. $2c + 11 = c + 44$
76. $53 = 44 + s$
77. $57 - x = 39$
78. $82 = -b + 33$
79. $x - 43 = 77$
80. $k + 89 = 56$
81. $3a + 8 = 2a - 5$
82. $43 = 91 + s$
83. $31 - x = 87$
84. $11 = -b + 28$
85. $x - 35 = 85$
86. $y - 23 = 63$
87. $2a + 9 = a + 39$
88. $65 = 7 + b$
89. $79 - x = 43$
90. $64 = -b + 65$
91. $f - 35 = 28$
92. $y + 36 = 90$
93. $4h + 32 = 3h + 45$
94. $13 = 54 + h$
95. $51 - x = 89$
96. $71 = -b + 19$
97. $x - 45 = 44$
98. $2y + 13 = 17$
99. $x - 13 = 20$
100. $y + 44 = 23$
101. $11a + 20 = 10a + 9$
102. $21 = 16 + s$
103. $43 - x = 18$
104. $9 = -b + 56$
105. $x - 9 = 8$
106. $y + 76 = 560$
107. $4a + 8 = 3a + 44$
108. $16 = 113 + s$
109. $4 - x = 40$
110. $400 = -b + 360$
111. $x - 34 = 55$
112. $y + 53 = 81$
113. $2a + 28 = 3a + 6$
114. $33 = 67 + s$
115. $45 - x = 72$
116. $91 = -b + 43$
117. $x - 76 = 96$
118. $y + 3 = 62$
119. $8b + 12 = 7b + 15$
120. $-21 = 45 + t$