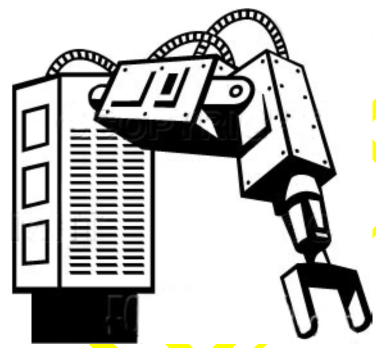
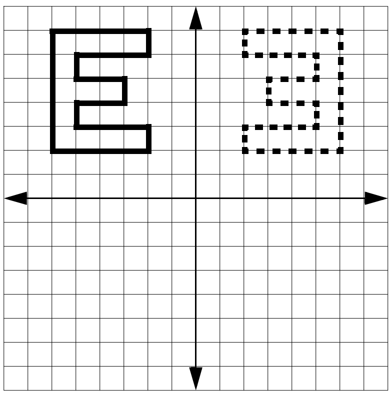
Section 12.6
Transformations: Reflections, Translations, and Rotations

Transformations are movements that take place in a graph and, in more advanced models, are the basis for the programming that controls robots for industry and entertainment.

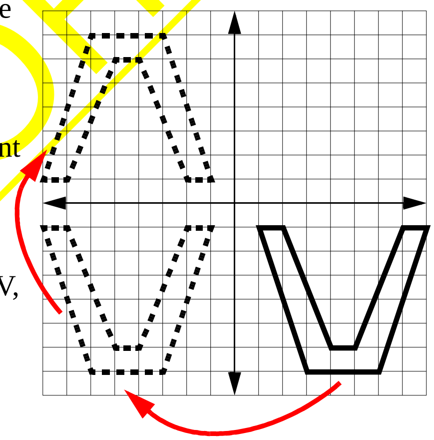


Industrial robot

REFLECTIONS



A reflection is what the word implies: A mirror image of a figure or design. The graph to the left shows the reflection of an image from quadrant II to quadrant I. Notice that reflections must take place over a particular axis and turn out to be reversed. This particular reflection is in respect to the *y*-axis; therefore, every corresponding point of the letter "E" in quadrant II is at the same distance from the *y*-axis in quadrant I.



Example: Find the reflection of the letter "V" starting in quadrant IV, with respect to the *y*-axis and then the *x*-axis.

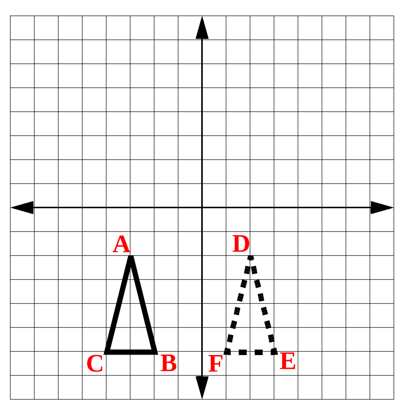
The solution can be seen in the graph to the right.

The points that define the letter "V" in the IV quadrant are, from left to right, (1,–1), (2,–1), (3,–7), (4,–6), (5,–6), (6,–7), (7,–1), and (8,–1). The reflection over the *y*-axis keeps the same values for *y*, but because the reflection is in over the *y*-axis, the *x* values change sign: (–1,–1), (–2,–1)...

The second reflection is going to quadrant II, where the *x* values are negative and the *y* values positive; therefore, again the values will be the same, except that the signs will be the opposite (–,+) to the original in quadrant IV.

TRANSLATIONS

Translations are movements which go UP, DOWN, RIGHT, and/or LEFT.



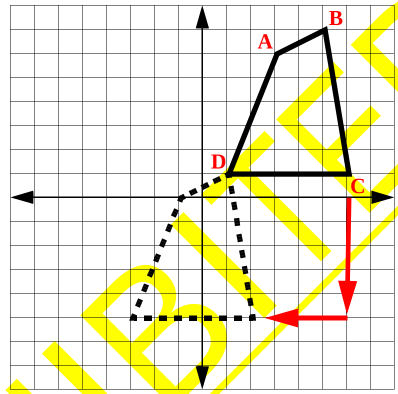
Example: In the graph to the right, make a translation of triangle (Δ)ABC five units to the right.

The original position of ΔABC was A(–3,–2), B(–2,–6), and C(–4,–6). Because it is to the right, all the *x* values of the coordinates change, but the *y* values stay the same. Simply add 5 to each *x*.

–3 + 5 = 2   –2 + 5 = 3   –4 + 5 = 1

The new coordinates are: D(2,–2), E(3,–6), and F(1,–6)

Example: In the graph to the right, make a translation of quadrilateral (▢)ABCD four units to the left and six units down ($x – 4$, $y – 6$).
(Left, Down)



Because the translation is both to the LEFT and DOWN, both coordinates will change.

The original figure is at: A(3,6), B(5,7), C(6,1), D(1,1)

Translation to the left (*x-coordinate*) –4:

3 – 4 = –1   5 – 4 = 1   6 – 4 = 2   1 – 4 = –3

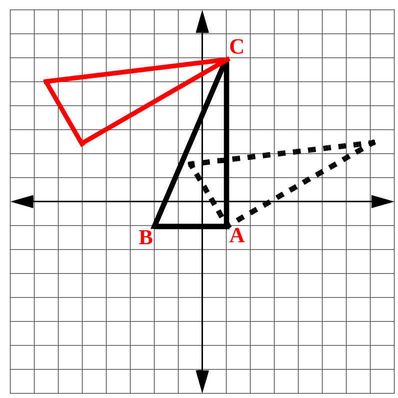
New coordinates after the first move: (–1,6), (1,7), (2,1), (–3,1)

Translation down (*y-coordinate*) –6:   6 – 6 = 0   7 – 6 = 1   1 – 6 = –5   1 – 6 = –5

New coordinates after the second move: (–1,0), (1,1), (2,–5), (–3,–5)

ROTATION

Measured in angles, rotation, also called *angular motion*, is the circling motion of a spinning object. All rotations must have a "center of rotation" about which the object moves.

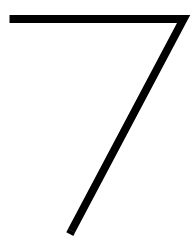


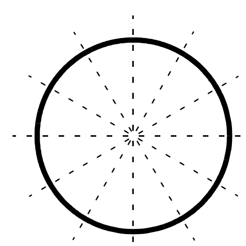
Example: In the graph to the right, rotate (rotation is always clockwise) the ΔABC 60º using point "A" as the center of rotation.

Because the rotation took place around point "A", point "A" did not move; however, both point "B" and "C" moved. In rotation, points farther away move more than points closer to the center of rotation; therefore, point "B" moved less than point "C." If the center of rotation changes, for example from "A" to "C", then the new location of the triangle will be completely different. (The red triangle above shows a rotation of 60º about "C")

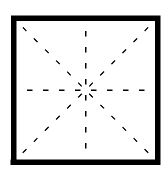
AXES (OR LINES) OF SYMMETRY

An object is said to have an axis of symmetry if, when folded about an axis, both sides of the fold match. In other words, there is "mirror symmetry" of half of the object. For example, a circle has an infinite number of axes of symmetry, while the number 7 has none.





Example: Find the number of axes of symmetry of a square.

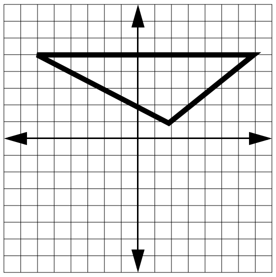


The number of lines are 4, which is the number of axes. (Horizontal, vertical, and both diagonals.

Practice:

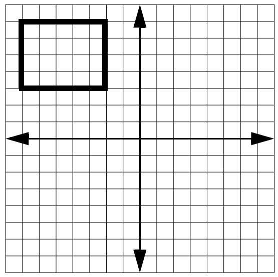
Determine the coordinates of the reflection described for each figure shown.

1.



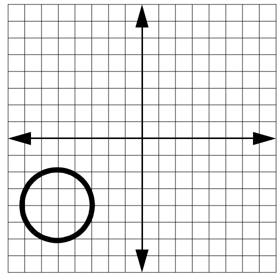
With respect to the *x*-axis

2.



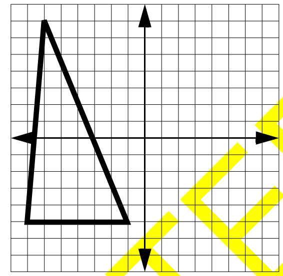
With respect to the *y*-axis

3.



With respect to the *x* and *y*-axis

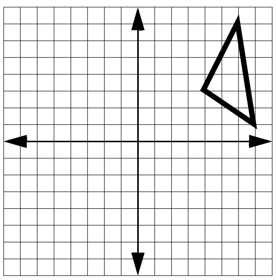
4.



With respect to the *y*-axis

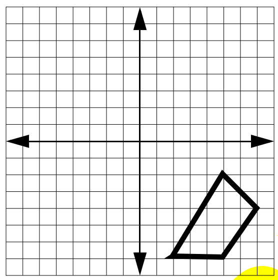
Make a translation of each figure below according to the instructions.

5.



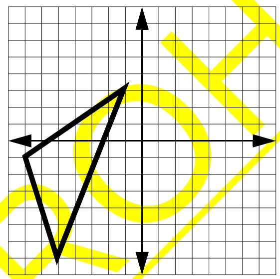
Five units to the left

6.



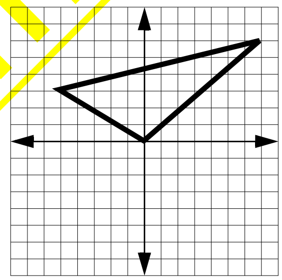
Six units to the left and seven units up.

7.



Eight units to the right

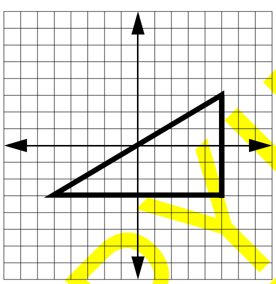
8.



Two units to the left and five units down.

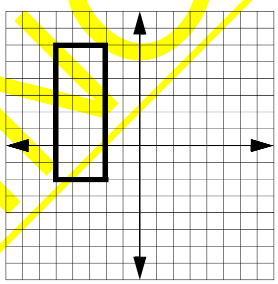
Copy graph, and using a protractor rotate each figure below according to the instructions.

9.



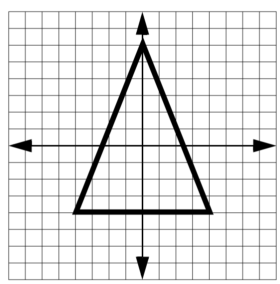
Rotate 20° using (5,3) as center

10.



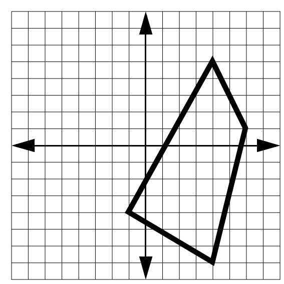
Rotate 90° using (3,–2) as center

11.



Rotate 45° using (0,6) as center

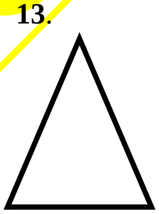
12.



Rotate 30° using (6,1) as center

Determine the number of lines of symmetry of each figure.

13.

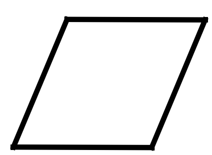


isosceles triangle

14.

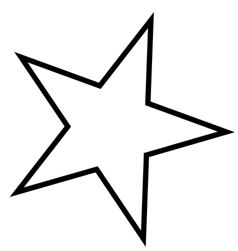
rectangle

15.



rhombus

16.



regular concave decagon