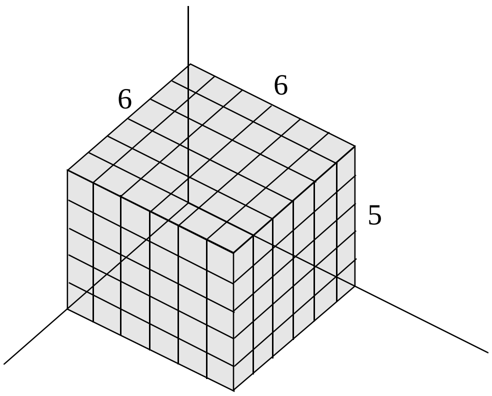
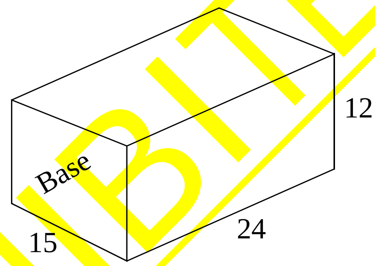
Section 12.5
Volume

Volume is the way *space* is measured. Space could be a room, a box, a warehouse, or the inside of an automobile engine. Volume is the product (multiplication) of quantities measured in THREE directions.



In the graph to the left, volume is the solid that has been darkened and it accounts for every one of the smaller cubes in the corner. In this particular example, the top layer has (6 × 6) = 36 cubes. Because there are 5 layers of 36 cubes each, the total volume is (36)(5) = 180.



Example: Find the volume, in inches, for the rectangular prism shown.

$\text{Volume}_{\text{prism}} = Bh$ Where: B = Area of base h = Height or length

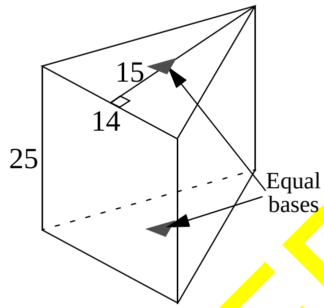
Volume = (15)(12)(24) = 4320

FORMULAS TO FIND THE VOLUME OF CERTAIN SHAPES

VOLUME FORMULAS

FIGURE	FORMULA	COMMENTS
Rectangular prism	Bh	Area of Base × Height or Length (Also for the cube.)
Triangular prism	$\frac{Bh}{2}$	The volume of any prism is the area of the cross-section of the prism times height or length.
Cylinder	$\pi r^2 h$	Where r is the radius, $\pi = 3.14$, and h the height or length.
Cone	$\frac{\pi r^2 h}{3}$	Volume of a cone is one-third the volume of a cylinder of the same height and diameter.
Pyramid	$\frac{Bh}{3}$	Volume of any pyramid is one-third the volume of a prism of the same height and base.
Sphere	$\frac{4}{3}\pi r^3$	Notice that the volume of the sphere involves the radius cubed (third power.)

Example: Find the volume, in cubic feet, of the triangular box shown.



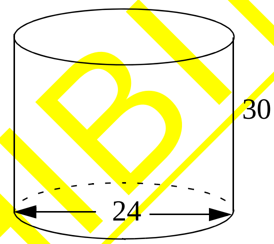
The box is in the shape of a triangular prism. Two *h*s are found: One for the triangular base (15), and the second one for the prism itself (25).

The area of the base (B) of the triangle is $\frac{bh}{2}$, where $b = 14$ and $h = 15$.

The h in the equation below refers to the height of the prism (25).

$$B = \frac{(14)(15)}{2} = 105 \qquad \text{Volume}_{\text{T. P.}} = \frac{Bh}{2} = \frac{(105)(25)}{2} = 2625 \text{ ft}^3$$

Example: Find the volume of the water tank shown if the height is 30 feet and the width 24 feet.



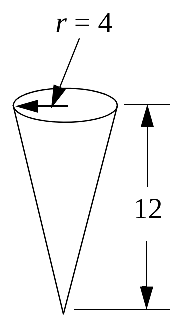
$\text{Volume}_{cy} = \pi r^2 h$ The width of a cylinder is the diameter.

Because the radius is needed, $r = \frac{24}{2} = 12$ feet

Base

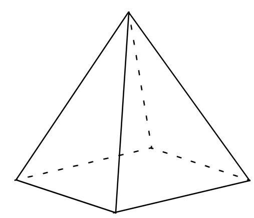
$$\text{Volume}_{cy} = \pi r^2 h = (3.14)(12)^2(30) = 13{,}564.8 \text{ ft}^3$$

Example: Find the volume of the ice cream cone shown if the radius is 4 centimeters and the height 12 centimeters.



$$\text{Volume}_{cone} = \frac{\pi r^2 h}{3} = \frac{3.14 \times 4^2 \times 12}{3} = 200.1 \text{ cm}^3 \text{ (rounded)}$$

Example: Find the volume of the pyramid shown if the height is 80 meters, and its base is 11,000 square meters.



$$\text{Volume}_{py} = \frac{Bh}{3} = \frac{11000 \times 80}{3} = 293{,}333.\overline{3} \text{ m}^3.$$

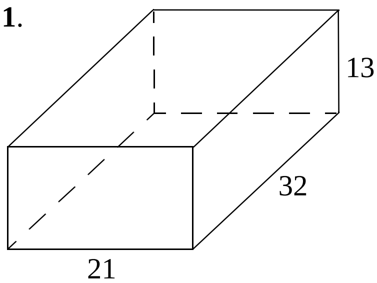
Example: Find the volume inside a basketball if the ball is 9 inches wide.

In a sphere, width means diameter. Because the radius is needed, divide: $r = \frac{d}{2} = \frac{9}{2} = 4.5$

$$\text{Volume}_{sphere} = \frac{4}{3}\pi r^3 = \frac{(4)(3.14)(4.5)^3}{3} = 381.51 \text{ in}^3.$$

Practice:
Find the volume.

1.



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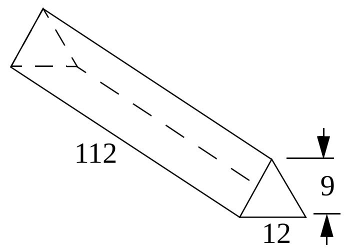

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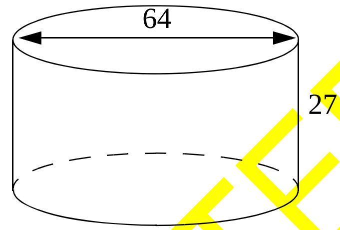

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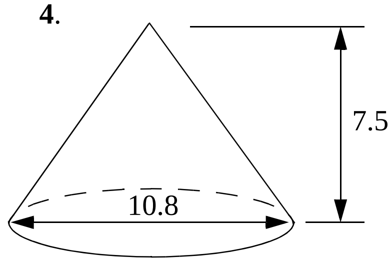

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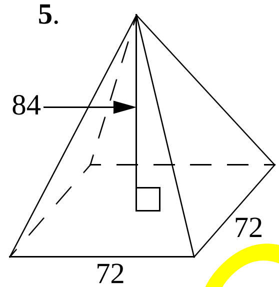

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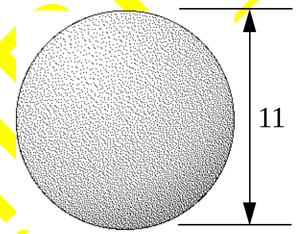
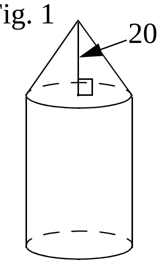


Solve.

7. Figure 1 is a silo to store grain. If the silo is 45 feet wide and 80 feet high, including cone, how much volume is available? (Notice that the silo is a composite shape made of a cone and a cylinder.)

Fig. 1



8. Figure 2 is a composite of a cube and a pyramid. Find the volume of both cube and pyramid if the pyramid is, like the cube, also 8 feet high.
9. A non-stretchable beach ball 22 inches wide needs to be filled with air. What volume of air will it take?
10. A rectangular container measures 20 feet in length and is 10 feet high. If the volume of the container is 3000 cubic feet, how wide is the container?
11. If you remove the volume of the cylinder from the cube in figure 3, how much volume is left?
12. How many boxes size "A" can you fit into box "B"? (Hint: draw it.)

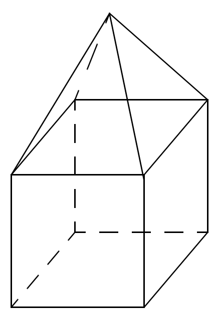
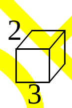
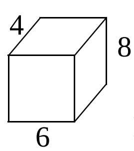


Fig. 2



A



B

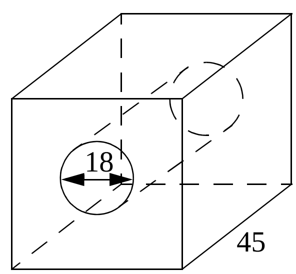


Fig. 3

13. Cylinder "C" is 48 inches wide and 64 inches long, while cylinder "D" is 12 inches wide and 16 inches high. How many size "D" cans will cylinder "C" fill?