Section 11.3
The Counting Principle, Permutations, and Combinations

THE COUNTING PRINCIPLE

When more than one event is taking place, the counting principle is used to determine the possible outcomes. The counting principle states that all the possible choices should be multiplied.

Example: A student has 7 skirts and 6 blouses in her closet that she may combine to produce one outfit. How many ways can she combine all the skirts and all the blouses?

The short answer would be: $7 \times 6 = 42$

In the long answer consider that the first skirt will match with 6 blouses, and the second skirt will do the same, and the third the same, and so on. Seven skirts matching 6 times each is 42.

Example: A printer has 16 colors and needs to select two of the colors for the cover of a book, one for the text and one for the background. In how many different ways can the cover be printed?

When the printer selects one color for background, there are 15 colors left for the text. If this is done for all 16 colors:

Each of the 16 colors, with 15 other selections: $16 \times 15 = 240$ possibilities

PERMUTATIONS

A permutation is when a set is rearranged—in other words, an event in which one thing is substituted for another.

Example: How many ways can the letters in the name CARLOS be arranged?

Trying it manually by moving letters around would take a long time; however, the following equation can be used to solve it

$P = n!$ Where $n!$ is n-factorial

A factorial is:

The product of all the positive integers from a given number to 1

Because CARLOS has 6 letters: $P = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways

COMBINATIONS

If order in selection is important, permutations is the way to solve a problem, but if order is NOT important, then the equation for combinations can be used:

$$C = \frac{n(n-1)(n-2)(n-3)\ldots}{k!}$$

Where ***n*** is the total number in the group taken "***k***" at a time. This way ***n*** is multiplied by the reduced "***k***" value in the numerator, then divided by "***k***" factorial. See example below.

By order it is meant that selecting, for example, three students for a task (k), the order could be Moe, Pat, and Jerry or Jerry, Moe, and Pat.

Example: In a group of 12 students, the teacher wants to make groups of three for a project. How many different groups of three could she make?

The question here is how many combinations are there in 12 items taken 3 at a time.

Reduced by "***k***" (3) is represented as 12, 11, 10

$$C = \frac{12(12-1)(12-2)}{3!} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = \frac{1320}{6} = 220$$

Example: In a group of 25 students, how many different groups of 6 could be made for a project?

$$C = \frac{25 \times 24 \times 23 \times 22 \times 21 \times 20}{6!} = \frac{127512000}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 177{,}100$$

Practice:

1. In her closet, a student has 3 skirts and 8 blouses that she may combine to produce one outfit. How many ways can she combine all the skirts and all the blouses?
2. A printer has 8 colors and needs to select two of the colors for the cover of a book, one for the text and one for the background. In how many different ways can the printer print the cover?
3. How many ways can the letters in the name CHARLES be arranged?
4. In a group of 9 students, the teacher wants to make groups of two for a project. How many different groups of two could she make?
5. In a group of 12 students, how many different groups of 4 could be made?
6. A car dealership offers in its convertible line five different exterior colors, three different tops, and four different interior colors. How many different ways can a customer combine the options?
7. A chef has five different salads: lettuce, carrot, watercress, cole slaw, and spinach. How many ways can he arrange them in a straight line on a serving table.
8. An eight-member band wants to pair musicians for different musical sets. How many different pairs could be formed?
9. A builder laying down a brick pattern for a driveway has five different colored bricks that come in two different shapes. How many ways could she combine the colors and the shapes of the bricks to form the pattern of the driveway?
10. A pool of ten workers is available for daily assignments. If the assignments are for three workers at a time, how many different groups of three workers could be formed from the ten workers?
11. A chef has six ingredients to make a stew. How many different stews could he make using three ingredients at a time?
12. Your e-mail password has eight letters. If someone wanted to steal your password, considering there are 26 letters in the alphabet, how many different attempts must the thief try? (Hint: How many different sets of eight out of a group of 26).
13. Your password, having eight different letters, may be rearranged by you. How many permutations could you form from those eight letters?
14. A basketball team is allowed to have twelve members on its roster. How many different teams of five may the coach form out of the twelve players?
15. How many ways can the letters in the name FLORIDA be arranged?