Section 11.2
Introduction to Probability

Probability is a ratio that shows the likelihood of an event taking place. It is the number of successful outcomes compared to the number of possible outcomes that could actually happen.

Because the probability of an event taking place for sure (100%) is 1 (for example, what is the probability of the sun rising tomorrow?), then anything less than 100% probability must be less than 1. In other words, a probability is a decimal, a percent, or a fraction that tells the chance of an event happening in the future.

Example: In a football game a referee tosses a coin to decide which team will kick first. What is the percent probability of your team kicking first?

Two teams, two faces to a coin.

Number of successful outcomes: 1

Number of possible events: 2 Probability = $p = \frac{1}{2} = 0.5 = 50\,\%$

Probability has a very wide use in industry as an applied and research tool. In the field of quality control, for instance, it is used to predict the number of failures.

SIMPLE PROBABILITY

The probability of an event which is always going to occur is 1; the probability of an event which never happens is 0. Furthermore, if the probability of an event is

p, then the probability of the same event not taking place is $(1 - p)$

Example: In a jar Paula has 10 red, 9 white, 3 blue, and 14 yellow jelly beans. What is the fraction probability of Paula picking a white jelly bean from the jar?

36 jelly beans in total (10 + 9 + 3 + 14).

Number of successful outcomes (white jelly beans): 9

Number of possible outcomes: 36

$$p = \frac{number\ \ of\ \ white}{total\ \ number\ \ of\ \ jelly\ \ beans} = \frac{9}{36} = \frac{1}{4}$$

Decimal probability $\frac{1}{4} = 0.25$ Percent probability = $0.25 \times 100 = 25\%$

COMPOUND PROBABILITY

A compound probability is when a probability is taken further. In other words, it is when more than one event is involved.

In the example above the probability could be compounded in two different ways:

1. By including the compound probability of another color.
2. By repeating the same color twice.

Number 1 situation above is a case of either one color OR the other. Because it is an increase, add.
Number 2 is a case of one color AND then another. Because it decreases, multiply.

Example: In a jar Raul has 10 red, 9 white, 3 blue, and 14 yellow jelly beans. What is the percent probability of Raul picking a white OR a blue jelly bean from the jar?

36 jelly beans in total (10 + 9 + 3 + 14).

Number of successful outcomes (white and blue jelly beans): 9 + 3 = 12

Number of possible outcomes: 36

$$p = \frac{white + blue}{total} = \frac{9+3}{36} = \frac{12}{36} = \frac{1}{3}$$

Decimal probability $\frac{1}{3} = 0.33$ Percent probability = $0.33 \times 100 = 33\%$

Example: In a jar Brittany has 10 red, 9 white, 3 blue, and 14 yellow jelly beans. What is the percent probability of Brittany picking a white first AND then a blue jelly bean from the jar *without replacing* the white one?

Without replacement

36 jelly beans in total (10 + 9 + 3 + 14).

Number of successful outcomes (white jelly beans): 9 $p = \frac{9}{36} = \frac{1}{4}$

Number of successful outcomes (blue jelly beans): 3

Because one white jelly bean has been eaten, the total number now is: 35 $p = \frac{3}{35}$

Multiply probabilities for white and blue jelly beans: $\frac{1}{4} \times \frac{3}{35} = \frac{3}{140} = 0.0214 \times 100 = 2.14\%$

With replacement

Suppose the white jelly bean was not to Brittany's liking and she puts the white jelly bean back inside the jar. Because the total number of beans would remain at 36, the probability—and answer—for the blue jelly bean will change.

Replacing white jelly bean $p = \frac{3}{36} = \frac{1}{12}$

Multiply new probabilities for white and blue jelly beans: $\frac{1}{4} \times \frac{1}{12} = \frac{1}{48} = 0.0208 \times 100 = 2.08\%$

Example: A group of 4 boys and 3 girls rush to the school cafeteria to have lunch. What is the percent probability that the first and last in line would be girls?

Number of successful outcomes (first in line a girl): 3 $\quad p = \frac{3}{7}$

Number of possible events: 7

Because one girl is already at the head of the line, subtract one girl from the group.

Number of successful outcomes (last in line a girl): 2 $\quad p = \frac{2}{6} = \frac{1}{3}$

Number of possible events: 6

Multiply probabilities for first *and* last a girl: $\frac{3}{7} \times \frac{1}{3} = \frac{3}{21} = 0.1428 \times 100 = 14.28\%$

Practice:
Solve

1. Five airplanes line up for taxing and take-off, two jets and three propeller-driven aircraft. If the planes take-off at random, what is the probability that the jets would depart one-two?
2. A bag of marbles contains 8 blue, 7 yellow, and 9 orange marbles. Find the percent probability of selecting two orange marbles in a row without looking? The first marble is not replaced.
3. Six men and five women are selected to form a line at random. What is the percent probability that the first three in line are men?
4. What is the probability that two blue marbles are selected at random from a bag that contains 20 red marbles and 10 blue marbles? The first blue marble is returned to the bag.
5. Five ducks, two brown and three black, land randomly on a pond. What is the compound probability that the brown ducks land one-two?
6. A group of five students, two boys and three girls, line up for lunch randomly. What is the percent probability that two of the girls will be first and last in line?
7. Find the percent probability of rolling "seven" two times in a row when throwing a pair of dice?
8. Thirty pebbles of three different colors are marked with numbers from 1 to 10 and held in a box. What is the percent probability that the same number is drawn at random from the box in three different colors?
9. Four types of soda held in a cooling tank are removed two at a time. If the cooler holds 120 cans and the number of soda cans are represented equally, what is the percent probability that someone would take two of the same type of soda at random?
10. In an area of the Florida Keys there are twice as many red snapper fish as there are grouper, and twice as many groupers as there are yellow tails. If you are fishing there and you catch a fish from a school of fish that contains all three types of fish mentioned, what is the percent probability of catching a red snapper, considering there is something on the line?
11. A deck of playing cards has two different colors and six different suits of 10 cards each. If four cards are dealt from the deck, what is the percent probability that all four are of the same color?
12. Six men and four women are selected to form a line at random. What is the percent probability that the first and fifth person in line are women?
13. A deck of playing cards has four cartoon characters, six musical instruments, and each comes in four different colors. What is the percent probability that the first and second card are musical instruments of the same color?