

Section 11.1

Measures of Central Tendencies and Spread

Understanding data is one of the objectives of *statistics*, which is the collection, organization, and interpretation of numerical data. Measuring the central tendencies and spread of data helps in the interpretation of the numbers collected.

MEAN

Popularly known as “average”, the mean of a set of data is found by *adding* and then *dividing*. It is computed by using the equation

$$MEAN = \frac{S}{n}$$

Where S = the sum of all numbers
 n = the number of observations

Example: Roberto's exam grades for a math class are as follows: 86, 73, 55, 96, 100, 75, 73, 66, 80, and 80. Find the mean.

First, organize data from lowest to highest

55, 66, 73, 73, 75, 80, 80, 86, 96, 100 Number of observations (exams): 10

Add all the numbers: $55 + 66 + 73 + 73 + 75 + 80 + 80 + 86 + 96 + 100 = 784$

$$MEAN = \frac{S}{n} = \frac{784}{10} = 78.4$$

Example: In the above example, what grade should Roberto earn on the final exam to increase the mean of his scores to 80?

The sum of his first 10 exams: 784

The sum of 10 exams plus final: $784 + x$

Where x is the desired grade for the final

New mean: 80

New number of exams, n : 11

$$80 = \frac{784 + x}{11}$$

$$80(11) = 784 + x$$

$$880 = 784 + x$$

$$880 - 784 = x$$

$$x = 96$$

Roberto must get at least **96** on the final to reach a mean of **80**.

MEDIAN

After organizing the data from lowest to highest, the value in the middle of the data is the *median*. Officially it is: *The middle value in a distribution, above and below which lie an equal number of values.*

Example: Find the median of the following observations: 30, 45, 22, 65, 45, 76, 3, 15, 88, 65, 44

First, organize data from lowest to highest:

3, 15, 22, 30, 44, **45**, 45, 65, 65, 76, 88

← 5 are lower 5 are higher →

Number of observations: 11

Because it is an odd number of observations, the median must be the 6th number (5 above and 5 below the median).

Answer: 45

Example: When the number of observations is even, the median is found by averaging the two closest numbers to the center. Find the median of:

86, 73, 55, 96, 100, 75, 73, 66, 80, and 80

First, organize data from lowest to highest:

55, 66, 73, 73, 75, 80, 80, 86, 96, 100 Number of observations: 10

The two closest numbers to the center are 75 and 80. Add both and divide by 2.

$$\frac{75 + 80}{2} = 77.5$$

MODE

The *mode* is: *The value occurring most frequently in a series of observations.*

Example: Find the mode in the following data: 4.5, 6.5, 7.2, 8.1, 6.5, 9.2, 6.6, 7.2, 5.5, 9.1, 7.2, 4.8

First, organize data from lowest to highest:

4.5, 4.8, 5.5, 6.5, 6.5, 6.6, 7.2, 7.2, 7.2, 8.1, 9.1, 9.2 Number of observations: 12

The value occurring most often is 7.2 (three times).

A set of data could have multiple modes (**bimodal**, **trimodal**...) Multiple modes have a negative effect on central tendencies.

CENTRAL TENDENCIES

A set of data is said to have central tendencies if the value of the mean, median, and mode are the same or nearly the same.

Example: Is the data in the example above displaying central tendencies?

4.5, 4.8, 5.5, 6.5, 6.5, 6.6, 7.2, 7.2, 7.2, 8.1, 9.1, 9.2

$$\text{Mean} = \frac{82.4}{12} = 6.9$$

$$\text{Median} = \frac{6.6 + 7.2}{2} = 6.9$$

$$\text{Mode} = 7.2$$

Answer: The mean and median show exact central tendencies, and the mode is off by only +0.3.
The data shows central tendencies.

Example: Does the data show central tendencies?

86, 73, 55, 96, 100, 75, 73, 66, 80, and 80

First, organize data from lowest to highest:

55, 66, 73, 73, 75, 80, 80, 86, 96, 100

Number of observations: 10

$$\text{Mean} = \frac{784}{10} = 78.4$$

$$\text{Median} = \frac{75 + 80}{2} = 77.5$$

Two Modes: 73, 80

Answer: The mean and median show central tendencies for the data, but the mode does not.

SPREAD

One way to measure data spread is by computing the **range**, and the range is found by subtracting the lowest value from the highest value. The spread illustrates how far the data reaches.

$$\text{Range} = \text{Highest value} - \text{Lowest value}$$

Examples: Find the range for the following sets of data:

55, 66, 73, 73, 75, 80, 80, 86, 96, 100

$$\text{Range} = 100 - 55 = 45$$

3, 15, 22, 30, 44, 45, 45, 65, 65, 76, 88

$$\text{Range} = 88 - 3 = 85$$

4.5, 4.8, 5.5, 6.5, 6.5, 6.6, 7.2, 7.2, 7.2, 8.1, 9.1, 9.2

$$\text{Range} = 9.2 - 4.5 = 4.7$$

Practice:

In the exercises below find the *mean*, *median*, *mode*, and *range*. State if *central tendencies* were found.

- Weight, in pounds, of different boxes: 23, 24, 25, 20, 21, 28, 18, 22, 20, 22, 20, 25.
- Height, in inches, of students in Ms. Llevada's class: 68, 60, 55, 62, 61, 62, 65, 66, 60, 63, 62.
- Distance, in feet, of home runs hit by a slugger: 420, 355, 360, 375, 400, 440, 388, 480, 342.
- Wind speed of a Caribbean hurricane: 75, 77, 78, 74, 76, 80, 78, 79, 88, 81, 78, 79, 79, 80, 78.
- Thickness, in inches, of water pump gaskets: 0.009, 0.01, 0.008, 0.011, 0.011, 0.008, 0.01, 0.008
- Output, in volts, of a certain electrical generator: 111, 110, 112, 110, 115, 116, 115, 110, 115, 112.
- Weight of beetles, in milligrams, for an experiment: 125, 120, 122, 121, 123, 121, 120, 121, 125.
- Speed, in miles per hour, at a car race: 198, 199, 201, 197, 202, 201, 199, 197, 199, 198, 201, 198.
- Closing price of crude oil: 62, 65, 62, 63, 71, 60, 59, 63, 62, 63, 62, 58, 60, 64, 62, 62, 63, 65, 60, 62.
- Daily temperature for New York: 67, 68, 67, 66, 69, 70, 68, 67, 67, 69, 60, 66, 67, 68, 67, 66, 65, 67.
- Rainfall, in inches, for Miami: 1.22, 1.25, 1.22, 1.2, 1.2, 1.23, 1.2, 1.24, 1.22, 1.22, 1.23, 1.22, 1, 1.3
- Noise level, in decibels, in a concert: 98, 99, 98, 97, 80, 99, 89, 98, 99, 98, 96, 100, 102, 99, 98, 97.
- On time performance for airport departures: 3, -5, -10, 1, 4, -8, -7, -12, 4, 5, -6, -9, 5, 1, 5, -3, -11.
- Fuel efficiency for small cars, in miles per gallon: 32, 28, 35, 33, 28, 26, 26, 28, 29, 30, 33, 30, 29, 28.
- Fuel efficiency for passenger cars, in miles per gallons: 12, 35, 22, 10, 15, 17, 28, 25, 38, 42, 14, 17.
- Weight of broiler chickens, in pounds: 3.25, 3.22, 3.55, 3.33, 3.35, 3.4, 3.2, 3.28, 3.25, 3.5, 3, 3.32.
- Speed gun error, in miles per hour: 0.5, -0.1, 0.6, -0.5, 0.7, -0.55, 0.4, -0.3, -0.1, -0.6, 0.4, -0.3, 0.2.