

Section 10.8

Complex Fractions

This a complex fraction:
$$\frac{8 + \frac{1}{x-1}}{\frac{5}{x+1} - 2}$$

Basically, these are large fractions made from fractions. To solve them, approach one section at a time.

THE NUMERATOR

The numerator of the “mother” fraction is an addition of fractions that reads: $\frac{8}{1} + \frac{1}{x-1}$
where the common denominator is $(x-1)$

Multiply left-hand fraction to adjust for the common denominator $\frac{8}{1} \times \frac{(x-1)}{(x-1)} = \frac{8x-8}{x-1}$

Add numerator fractions:
$$\frac{8x-8}{x-1} + \frac{1}{x-1} = \frac{8x-8+1}{x-1} = \frac{8x-7}{x-1} \quad (1)$$

THE DENOMINATOR

These denominator fractions are performing subtraction $\frac{5}{x+1} - \frac{2}{1}$

Multiply right-hand fraction to adjust for the common denominator: $\frac{2}{1} \times \frac{(x+1)}{(x+1)} = \frac{2x+2}{x+1}$

Subtract denominator fractions:
$$\frac{5}{x+1} - \frac{2x+2}{x+1} = \frac{5-2x-2}{x+1} = \frac{-2x+3}{x+1} \quad (2)$$

Dividing numerator and denominator

Divide “mother” fraction (results from numerator (1) over results of denominator (2): $\frac{8x-7}{x-1} \div \frac{-2x+3}{x+1}$

multiply by the reciprocal
$$\frac{8x-7}{x-1} \times \frac{x+1}{-2x+3} = \frac{(8x-7)(x+1)}{(x-1)(-2x+3)} = \frac{8x^2+x-7}{-2x^2+5x-3}$$

Example: simplify
$$\frac{\frac{2a}{5} + \frac{4a}{5}}{\frac{3a}{8}}$$

Numerator first: $\frac{2a}{5} + \frac{4a}{5} = \frac{2a+4a}{5} = \frac{6a}{5}$

Divide results of numerator by denominator: $\frac{6a}{5} \div \frac{3a}{8} = \frac{6a}{5} \times \frac{8}{3a} = \frac{48a}{15a} = \frac{16}{5}$

Example: Simplify $\frac{3 - \frac{1}{y}}{9 - \frac{1}{y^2}}$

Numerator first: $\frac{3}{1} - \frac{1}{y} = \frac{3y-1}{y}$

Then denominator: $\frac{9}{1} - \frac{1}{y^2} = \frac{9y^2-1}{y^2}$

Divide numerator by denominator. (Notice factoring of difference of squares).

$$\frac{3y-1}{y} \div \frac{9y^2-1}{y^2} = \frac{3y-1}{y} \times \frac{y^2}{9y^2-1} = \frac{y^2(3y-1)}{y(3y+1)(3y-1)} = \frac{y}{3y+1}$$

Practice:
Simplify.

1. $\frac{\frac{3}{4} + \frac{5}{4}}{\frac{1}{7}}$

6. $\frac{\frac{1}{ab}}{\frac{1}{a} + \frac{1}{b}}$

11. $\frac{\frac{a}{b} + \frac{b}{c}}{\frac{c}{d} - \frac{d}{e}}$

16. $\frac{\frac{x^2}{x^2+xy+x+y}}{\frac{x}{x+y}}$

2. $\frac{8 + \frac{12}{7}}{\frac{6}{11}}$

7. $\frac{\frac{3}{x+y}}{\frac{7}{x^2-y^2}}$

12. $\frac{12 + \frac{4}{y-8}}{\frac{5}{y+8} - 6}$

17. $\frac{\frac{1}{(h^2-j^2)}}{\frac{1}{(h+j)^2}}$

3. $\frac{\frac{18}{4} + \frac{22}{9}}{\frac{5}{12}}$

8. $\frac{\frac{a}{b} + \frac{b}{a}}{\frac{1}{b} - \frac{1}{a}}$

13. $\frac{15 - \frac{5}{x}}{8 - \frac{6}{x^2}}$

18. $\frac{a + \frac{1}{a-1}}{\frac{a}{a-1} - a}$

4. $\frac{\frac{a}{5} + \frac{15}{7}}{\frac{8a}{9}}$

9. $\frac{\frac{9}{c} + \frac{1}{4c}}{c + \frac{c}{9}}$

14. $\frac{\frac{5}{x} + \frac{7}{x^2}}{\frac{9}{x} - \frac{16}{3}}$

19. $\frac{\frac{x}{y} + \frac{2-x}{y+1}}{\frac{-x}{y+1} + \frac{x+3}{y}}$

5. $\frac{\frac{3}{5} + \frac{6}{x-2}}{\frac{4}{x-2} + 9}$

10. $\frac{\frac{w}{w-z}}{\frac{w^2}{w^2-z^2}}$

15. $\frac{\frac{8a}{4} + \frac{9a}{5}}{\frac{14a}{3}}$

20. $\frac{\frac{a+3}{a+5} + \frac{1}{a}}{\frac{a}{a+5} + \frac{1}{a}}$