

Section 10.7

Rational Equations

A rational equation is one that contains fractions. Because fractions are divisions, one simple way to start the solution of rational equations is by the use of multiplication. This will make for a less complicated solution as the fractions are eliminated.

Example: Solve $\frac{2}{5} + \frac{7}{2} = \frac{y}{10}$

The denominators show 10 is the common denominator for the equation; therefore, **multiplying each term** of the equation by 10 will eliminate the rational numbers (fractions).

$$\frac{(10)2}{5} + \frac{(10)7}{2} = \frac{(10)y}{10} \quad \text{reducing each term:} \quad 4 + 35 = y \quad \longrightarrow \quad y = 39$$

Example: Solve $y - \frac{5}{y} = -4$

This is a quadratic equation:
Write equal to zero, factor,
and solve.

Multiply every term by the common denominator y : $(y)y - \frac{(y)5}{y} = (y)-4 \quad \longrightarrow \quad y^2 - 5 = -4y$

solve for y : $y^2 - 5 = -4y \quad \longrightarrow \quad y^2 + 4y - 5 = 0 \quad \longrightarrow \quad (y + 5)(y - 1) = 0$

Answer: $y = -5, 1$

Example: Solve $\frac{5}{y+2} = \frac{12}{y-2}$

The denominators are not the same, even if it is only a sign difference. The common denominator is $(y + 2)(y - 2)$. Multiply **both** sides by $(y + 2)(y - 2)$.

$$\frac{(y+2)(y-2)5}{y+2} = \frac{(y+2)(y-2)12}{y-2}$$

reduce to: $(y-2)5 = (y+2)12$

distribute: $5y - 10 = 12y + 24$

solve for y : $-10 - 24 = 12y - 5y$

$$-34 = 7y$$

$$y = -\frac{34}{7}$$

Example: Solve $\frac{1}{(x+5)} + \frac{3}{(x-5)} = \frac{x^2-9}{x^2-25}$

The common denominator is $(x+5)(x-5)$, which happens to be also the difference of squares x^2-25 .
Multiply all three terms by $(x+5)(x-5)$ and reduce:

$$\frac{(x+5)(x-5)1}{(x+5)} + \frac{(x+5)(x-5)3}{(x-5)} = \frac{(x+5)(x-5)x^2-9}{x^2-25} \quad (x-5)1 + (x+5)3 = x^2-9$$

distribute: $x-5+3x+15 = x^2-9$ combine: $0 = x^2-4x-19$

Solve for x by completing the square: $x^2-4x+4 = 19+4 \rightarrow \sqrt{(x-2)^2} = \sqrt{23}$

$$x-2 = \sqrt{23} \rightarrow x = 2 \pm \sqrt{23}$$

Practice:

Solve.

1. $\frac{3}{4} + \frac{1}{3} = \frac{x}{12}$
2. $\frac{4}{9} + \frac{2x}{6} = \frac{x}{3}$
3. $\frac{2}{5x} + \frac{7}{x} = 1$
4. $\frac{8a}{5} + \frac{12a}{20} = \frac{a}{4}$
5. $\frac{5}{4a} + \frac{1}{3} = \frac{4}{9}$
6. $\frac{7}{6x} + \frac{1}{5x} = \frac{1}{15}$
7. $\frac{2}{x-1} + \frac{7}{x+1} = 1$
8. $\frac{x+1}{4} + \frac{x+2}{5} = 1$
9. $\frac{y-1}{3} + \frac{y-2}{2} = \frac{3}{4}$
10. $\frac{y+2}{7} - \frac{y-5}{14} = \frac{1}{2}$
11. $\frac{1}{y^2-9} - \frac{5}{(y-3)} = 1$
12. $z - \frac{12}{z} = \frac{1}{z}$
13. $\frac{y-3}{y+4} = \frac{y-5}{y+1}$
14. $\frac{2y-5}{y+3} = \frac{12}{y-8}$
15. $\frac{7a-1}{3a+5} = \frac{4a+3}{3a-5}$
16. $\frac{7}{x-3} - \frac{8}{x+3} = \frac{-4}{5}$
17. $\frac{y}{8} - \frac{5y}{24} = \frac{1}{6}$
18. $\frac{14}{x+8} - \frac{3}{x-4} = 1$
19. $\frac{15}{y+1} = \frac{14}{y-7}$
20. $\frac{5}{4(y+1)} = \frac{12}{2(y+1)}$
21. $\frac{24}{9(c-2)} = \frac{2c}{18}$
22. $\frac{5(y+3)}{3y+1} = \frac{2(y-5)}{3y-1}$
23. $\frac{p-3}{2p-1} = \frac{p+10}{2p+1}$
24. $\frac{-4}{z-1} + \frac{-z-5}{z+1} = 1$
25. $\frac{8}{x-4} + \frac{12}{x+4} = 5$
26. $\frac{1}{x-8} - \frac{1}{x+8} = \frac{-1}{2}$
27. $\frac{(y+5)}{4y+3} = \frac{(y-8)}{4y-3}$
28. $\frac{7}{x+1} - \frac{14}{x-1} = \frac{x^2-4}{x^2-1}$
29. $\frac{12}{2x-7} + \frac{6}{2x+7} = \frac{4x-9}{4x^2-49}$
30. $\frac{16}{x+8} - \frac{11}{x-8} = \frac{x^2+25}{x^2-64}$
31. $\frac{15}{x+4} + \frac{60}{x^2-16} = \frac{x+2}{x-4}$
32. $\frac{10}{2x+3} + \frac{9}{2x-3} = 1$
33. $\frac{18}{x^2-49} - \frac{20}{x-7} = \frac{1}{x+7}$
34. $\frac{7}{x^2-x-2} = \frac{8}{x^2-2x-3}$
35. $\frac{x-2}{x^2+5x+4} = \frac{x+3}{x^2+6x+8}$