Section 10.6
Addition and Subtraction of Rational Expressions with Different Denominators

Fractions with different denominators cannot be added nor subtracted. To do so we must change them.

Example: Add $\frac{x+4}{3x} + \frac{x-4}{x}$

The denominators show x is common, but 3 is not. Correct by multiplying second fraction times $\frac{3}{3}$ (one).

$$\frac{x-4}{x} \times \frac{3}{3} = \frac{3x-12}{3x}$$

Complete addition using altered rational expression

$$\frac{x+4}{3x} + \frac{3x-12}{3x} = \frac{x+4+3x-12}{3x} = \frac{4x-8}{3x}$$

Example: Add $\frac{5}{y+2} + \frac{12}{y-2}$

The denominators are not the same, even if it is only a sign difference. The common denominator is $(y + 2)(y - 2)$.

Multiply both fractions by one using the common denominator $\frac{(y+2)(y-2)}{(y+2)(y-2)}$

$$\frac{5}{y+2} \times \frac{(y+2)(y-2)}{(y+2)(y-2)} = \frac{5(y-2)}{(y+2)(y-2)} \quad (1)$$

$$\frac{12}{y-2} \times \frac{(y+2)(y-2)}{(y+2)(y-2)} = \frac{12(y+2)}{(y+2)(y-2)} \quad (2)$$

Add the results (1) and (2)

$$\frac{5(y-2)+12(y+2)}{(y+2)(y-2)} = \frac{5y-10+12y+24}{(y+2)(y-2)}$$

$$= \frac{17y+14}{(y+2)(y-2)} = \frac{17y+14}{y^2-4}$$

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Example: Subtract $\dfrac{x+5}{x^2+6x+9} - \dfrac{4}{x+3}$

Factoring the first denominator: $(x+3)(x+3)$

Because the second denominator is part of the second denominator, the first denominator is the *common denominator.*

Multiply the second fraction by **1** to get common denominator $(x+3)(x+3)$

$$\frac{4}{x+3} \times \frac{(x+3)}{(x+3)} = \frac{4x+12}{(x+3)(x+3)}$$

Subtract this result from the first fraction

write parenthesis for subtraction

$$\frac{x+5}{(x+3)(x+3)} - \frac{4x+12}{(x+3)(x+3)} = \frac{x+5-(4x+12)}{(x+3)(x+3)}$$

$$= \frac{x+5-4x-12}{(x+3)(x+3)}$$

$$= \frac{-3x-7}{(x+3)(x+3)} = \frac{-3x-7}{(x+3)^2}$$

Example: Subtract $\dfrac{-4}{x^2+6x+8} - \dfrac{-8}{x^2+8x+12}$

Factoring both denominators: $(x+2)(x+4)$ $(x+2)(x+6)$

Because $(x+2)$ is common to both, the common denominator is $(x+2)(x+4)(x+6)$

Multiply both fractions by **1** to get the common denominator

$$\frac{-4}{(x+2)(x+4)} \times \frac{(x+2)(x+4)(x+6)}{(x+2)(x+4)(x+6)} = \frac{-4(x+6)}{(x+2)(x+4)(x+6)}$$

$$\frac{-8}{(x+2)(x+6)} \times \frac{(x+2)(x+4)(x+6)}{(x+2)(x+4)(x+6)} = \frac{-8(x+4)}{(x+2)(x+4)(x+6)}$$

Subtract the results of the second fraction from the results of the first fraction:

$$\frac{-4(x+6)}{(x+2)(x+4)(x+6)} - \frac{-8(x+4)}{(x+2)(x+4)(x+6)} = \frac{-4(x+6)-[(-8)(x+4)]}{(x+2)(x+4)(x+6)}$$

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$$= \frac{-4x - 24 + 8x + 32}{(x+2)(x+4)(x+6)}$$

$$= \frac{4x + 8}{(x+2)(x+4)(x+6)}$$

$$= \frac{4(x+2)}{(x+2)(x+4)(x+6)}$$

$$= \frac{4}{(x+4)(x+6)} = \frac{4}{x^2 + 10x + 24}$$

Practice:
Add or subtract.

1. $\frac{x}{x^2 - 9} + \frac{3x}{x+3}$
2. $\frac{5a}{a^2 - 25} + \frac{2a}{a-5}$
3. $\frac{-2}{x-5} - \frac{10}{3x-15}$
4. $\frac{-5}{4p+24} + \frac{9p}{p+6}$
5. $\frac{2r}{(r+3)^2} - \frac{7r}{r+3}$
6. $\frac{-12}{(w-7)^2} - \frac{20}{w-7}$
7. $\frac{8x}{8x-24} + \frac{9x}{x-3}$
8. $\frac{8y}{12y+3} - \frac{5y}{4y+1}$
9. $\frac{2}{x^2+5x+6} + \frac{x}{x^2+4x+4}$
10. $\frac{4}{x^2-3x+2} - \frac{-15}{x^2+3x-10}$
11. $\frac{-6+5x}{2x^2} + \frac{-4x+3}{7x}$
12. $\frac{-y^2-5y+7}{9y} - \frac{y^2+6}{3y}$
13. $\frac{-8+2x}{12x^2} + \frac{3(5+x)}{x^2}$
14. $\frac{12y-1}{3y^2} - \frac{3y-12}{8y}$
15. $\frac{u-7}{4-u} + \frac{7u+10}{u+4}$
16. $\frac{-10}{(h+2)^2} + \frac{4h-8}{h+2}$
17. $\frac{x-1}{x^2-y^2} + \frac{y+1}{x^2-y^2}$
18. $\frac{-3(t+1)}{t^2-16} - \frac{t-1}{t+4}$
19. $\frac{-9}{x^2+x-20} - \frac{-7}{x^2+7x+10}$
20. $\frac{4a-2}{a-b} - \frac{3b+9}{a+b}$
21. $\frac{-2}{16b^2-8b+1} + \frac{11}{4b-1}$
22. $\frac{-1}{x^2-14x+49} + \frac{x-2}{x-7}$
23. $\frac{-15a}{a^2-b^2} - \frac{4b}{a-b}$
24. $\frac{-(y-1)}{y^2-5y+6} + \frac{y-5}{y^2-6y+8}$
25. $\frac{-8x}{x^2-25} - \frac{3x-2}{x-5}$
26. $\frac{-(c-2)}{18c-42} + \frac{c+1}{3c-7}$
27. $\frac{-(s+2)}{s^2-9} + \frac{2s-5}{9-s^2}$
28. $\frac{7}{y^2+6y+5} - \frac{y}{y^2+4y-5}$
29. $\frac{-(4x-1)}{2x-5} - \frac{-(x+3)}{2x+5}$
30. $\frac{5b}{b^2-16b+64} + \frac{-b}{b-8}$
31. $\frac{a-1}{a+1} - \frac{a+1}{a-1}$
32. $\frac{x+1}{x-1} + \frac{x+2}{x^2+2x-3}$
33. $\frac{-(x-1)}{x+1} + \frac{-(x+1)}{x-1}$
34. $\frac{-x}{x+5} - \frac{x+1}{x^2+10x+25}$
35. $\frac{-6x}{7x-9} + \frac{-5x}{7x+9}$
36. $\frac{x^2+5x+6}{(x+1)^2} - \frac{x^2-5x+6}{x^2+2x+1}$
37. $\frac{x+2}{x+2} - \frac{x+3}{x^2-4}$
38. $\frac{-(y-1)}{y^2-25} - \frac{-y}{y-5}$
39. $\frac{x+10}{x+8} - \frac{x+1}{x^2-64}$

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