

## Section 10.4

# Dividing Polynomials

### DIVIDING BY A MONOMIAL

**Example:** Divide  $\frac{8x^2 + 9x - 3}{4}$  Each term of the numerator is divided by 4

$$\frac{8x^2}{4} = 2x^2 \quad \frac{9x}{4} = 2.25x \quad \frac{-3}{4} = -0.75$$

Answer:  $2x^2 + 2.25x - 0.75$

**Example:** Divide  $\frac{9x^4 - 12x^3 - 14x}{3x}$

$$\frac{9x^4}{3x} = 3x^3 \quad \frac{-12x^3}{3x} = -4x^2 \quad \frac{-14x}{3x} = -4.\bar{6}$$

Answer:  $3x^3 - 4x^2 - 4.\bar{6}$

### DIVIDING BY A BINOMIAL

Division of a polynomial by a binomial is set up and computed similarly to regular division, except that here we use terms with base, coefficient, and exponent. In other words, each term will be divided, then multiplied and subtracted. Sometimes there will be a remainder, sometimes the last operation will leave none.

**Example:** Divide  $(y^2 + 8y - 10) \div (y - 2)$

$y - 2 \overline{) y^2 + 8y - 10}$  Divide the leading term of the trinomial by the leading term of the binomial:  $\frac{y^2}{y}$  and place the result above the line like regular division.

$y - 2 \overline{) y^2 + 8y - 10}$  Like in regular division, multiply  $y \cdot y$  and place it below the leading term of the trinomial.

$y - 2 \overline{) y^2 + 8y - 10}$  Do the same for the second term of the binomial,  $y \cdot -2$

$$\begin{array}{r} y \\ y-2 \overline{) y^2 + 8y - 10} \\ \underline{-(y^2 - 2y)} \\ 10y \end{array}$$

Now subtract. The first term of the trinomial cancels and the second term becomes 10y. This is the end of the first cycle.

$$\begin{array}{r} y \\ y-2 \overline{) y^2 + 8y - 10} \\ \underline{-(y^2 - 2y)} \\ 10y - 10 \end{array}$$

To start the second cycle, like in regular division bring down -10 and place it next to 10y.

$$\begin{array}{r} y + 10 \\ y-2 \overline{) y^2 + 8y - 10} \\ \underline{-(y^2 - 2y)} \\ 10y - 10 \end{array}$$

Divide  $\frac{10y}{y}$  and place answer above next to y and multiply y - 2 by 10.

$$\begin{array}{r} y + 10 \\ y-2 \overline{) y^2 + 8y - 10} \\ \underline{-(y^2 - 2y)} \\ 10y - 10 \\ \underline{-(10y - 20)} \\ 10 \end{array}$$

Multiply the second term, 10, of the answer by both terms of the binomial (y - 2) and place under (10y - 10) and subtract.

10 remainder

Answer:  $y + 10$  with  $\frac{10}{y-2}$  remaining

**Example:** Divide  $(x^3 + 1) \div (x + 1)$

$x + 1 \overline{) x^3 + 1}$  To properly divide an expression with missing terms like the binomial  $x^3 + 1$ , the middle terms  $x^2$  and  $x$  must be inserted. This is done by leaving space for the missing terms before beginning.

$$x + 1 \overline{) x^3 \quad \quad + 1}$$

$$\begin{array}{r} x^2 \\ x + 1 \overline{) x^3 \quad \quad + 1} \\ \underline{-(x^3 + x^2)} \\ -x^2 \end{array}$$

End of first cycle

$$\begin{array}{r} x^2 - x \\ x + 1 \overline{) x^3 \quad \quad + 1} \\ \underline{-(x^3 + x^2)} \\ -x^2 \quad \quad \quad \\ \underline{-(-x^2 - x)} \\ x \end{array}$$

End of second cycle

$$\begin{array}{r}
 x^2 - x + 1 \\
 x + 1 \overline{) x^3 \phantom{+ 0x^2 + 0x + 0} + 1} \\
 \underline{-(x^3 + x^2)} \phantom{+ 0} \\
 -x^2 \phantom{+ 0x + 0} \\
 \underline{-(-x^2 - x)} \phantom{+ 0} \\
 x + 1 \\
 \underline{-(x + 1)} \\
 0
 \end{array}$$

← Answer

End of third cycle. No remainder

**Practice:**

Divide.

1.  $\frac{20x^4 - 4x^3 + x^2 - 8x + 16}{4}$
2.  $\frac{y - 5y^2 + y^5}{y}$
3.  $\frac{16x^4 - 3x^3 + x^2 + 6}{6}$
4.  $\frac{40a^5 - 8a^3 + a^2}{a}$
5.  $\frac{18x^3 + 24x^2 + 6x}{3x}$
6.  $\frac{35y^3 + 15y^2 - 30y}{5y}$
7.  $\frac{30a^6 - 10a^4 + 5a^2}{-5a}$
8.  $\frac{24m^5 - 30m^4 + 6m^2}{-6m^2}$
9.  $\frac{20s^2t^2 + 5s^2t - 15s^2}{5st}$
10.  $\frac{15t^6 - 30t^4 + 6t^3}{3t^2}$
11.  $\frac{14c^2d^2 - 4c^2d + cd^2}{2c^2d}$
12.  $\frac{12a^4 - 24a^3 + 16a^2 - 8a}{-8a}$
13.  $\frac{24w^3z^3 + 16w^2z^2 - 8wz}{8wz}$
14.  $\frac{12y^2 - 9y + 15}{-3y}$
15.  $\frac{x^2 - 5x + 10}{x - 1}$
16.  $\frac{y^2 + 6y - 16}{y - 2}$
17.  $\frac{x^2 + 6x + 9}{x + 3}$
18.  $\frac{t^2 + 13t + 14}{t + 1}$
19.  $\frac{y^2 + 10y + 25}{y + 5}$
20.  $\frac{x^2 - 16}{x - 4}$
21.  $\frac{t^2 - 17t - 60}{t + 3}$
22.  $\frac{x^3 - x^2 + 5x - 25}{x - 5}$
23.  $\frac{m^3 + 2m^2 - 8m - 16}{m + 2}$
24.  $\frac{x^4 + 1}{y + 1}$
25.  $\frac{n^4 + 4n^3 - 2n - 8}{n + 4}$
26.  $\frac{t^4 - 1}{t - 1}$
27.  $\frac{2s^4 - 4s^3 + 6s^2 - s + 2}{s - 2}$
28.  $\frac{10t^3 + 20t^2 - 15t + 25}{t + 5}$
29.  $\frac{x^4 - x^3 + x^2 - x + 1}{x + 1}$
30.  $\frac{6y^4 + 12y^2 - 18y + 30}{3y + 2}$
31.  $\frac{x^3 + x^2 - 9x + 15}{x - 3}$
32.  $\frac{5t^3 + 10t^2 - 15t + 20}{5t - 1}$
33.  $\frac{25x^3 + 20x^2 - 10x + 5}{x - 5}$
34.  $\frac{4x^3 - 8x^2 - 12x + 16}{x + 4}$
35.  $\frac{30t^3 + 24t^2 - 12t + 6}{2t - 3}$
36.  $\frac{5x^3 + 10x^2 - 15x + 20}{x + 5}$
37.  $\frac{10m^3 + 8m^2 + 6m + 4}{m - 2}$
38.  $\frac{60y^3 - 15y^2 + 30y + 45}{5y - 3}$
39.  $\frac{8n^3 + 2n^2 - 8n + 4}{n - 4}$
40.  $\frac{12t^3 + 9t^2 - 6t + 3}{t + 3}$
41.  $\frac{2s^3 + 4s^2 - 6s + 8}{s + 2}$