

Section 10.1

Rational Expressions

Rational expressions are formed when we put polynomials in a fraction. In other words, rational expressions represent division.

Example: Simplify $\frac{x^2 + 3x - 10}{x^2 + x - 20}$

Because x^2 , x , 10 and 20 are “tied up” in the trinomials through addition and subtraction, on first view it seems impossible that anything could be reduced, or simplified. However, the idea in rational expressions is to find ways to reduce the fraction by factoring. The above rational expression can be factored to:

$$\frac{(x+5)(x-2)}{(x-4)(x+5)}$$

The move is obvious. Now $\left(\frac{x+5}{x+5} = 1\right)$ can be cancelled and the rational expression becomes

$$\frac{x-2}{x-4}$$

Example: Simplify $\frac{7x+14}{7x}$

$$\frac{7(x+2)}{7x}$$

$$\frac{x+2}{x}$$

Example: Simplify $\frac{y-5}{5-y}$

Because the expressions are not exactly the same, they cannot be cancelled. However, $5-y$ is the opposite of $y-5$ and it may be rewritten by factoring -1 . Therefore,

$$(5-y) \text{ turns into } -(-5+y) \text{ or } -(y-5)$$

And the new ratio is $\frac{y-5}{-(y-5)} = -1$ and can be reduced to -1 .

Example: Simplify $\frac{x^2 - 9}{x^2 + 7x + 12}$

$$\frac{(x-3)(x+3)}{(x+4)(x+3)} = \frac{(x-3)}{(x+4)}$$

Example: Simplify $\frac{7-2c}{8(2c-7)}$

Factor -1 from the numerator $\frac{-(-7+2c)}{8(2c-7)} = \frac{-(2c-7)}{8(2c-7)} = -\frac{1}{8}$

Practice:
Simplify.

1. $\frac{8x^6y^3}{24x^4y^4}$

2. $\frac{-35x^4y^7z^3}{25x^5y^4z}$

3. $\frac{9y+9}{36y}$

4. $\frac{-24y-48}{12y}$

5. $\frac{5a^4+5a^3}{10a^3+10a^2}$

6. $\frac{15x^2+45x}{10x-30}$

7. $\frac{8a^4+8a^3}{a^3+a^2}$

8. $\frac{12-6x}{x-2}$

9. $\frac{(x+y)^2}{(x+y)}$

10. $\frac{8x^3+16x^2-24x}{x^2+2x-3}$

11. $\frac{13y^4+52}{13(y+2)}$

12. $\frac{m-10}{10-m}$

13. $\frac{66x-6}{11x-1}$

14. $\frac{(p-4)(p+5)}{5+p}$

15. $\frac{9x^3+15x^2}{5+3x}$

16. $\frac{60a^{12}b^{15}c^{12}}{5a^{12}b^{14}c^9d^2}$

17. $\frac{7x}{49x+28x^2-14x^3}$

18. $\frac{4x^2-2x}{5x^2-5x}$

19. $\frac{6p^5-p^4}{p^2-p}$

20. $\frac{h^6-h^5}{h^8-h^7}$

21. $\frac{x^2-9}{x^2+5x+6}$

22. $\frac{a^2-5a-24}{a^2-3a-40}$

23. $\frac{x^2+x-2}{x^2-x-6}$

24. $\frac{y^2-y-72}{x^2-4x-96}$

25. $\frac{x^2-1}{x^2+2x-3}$

26. $\frac{2b^2-7b-15}{4b^2-14b-30}$

27. $\frac{3a^2+31a+36}{6a^2+62a+72}$

28. $\frac{x^2+8x+16}{x^2+2x-8}$

29. $\frac{y^2-5y-50}{y^2-8y-20}$

30. $\frac{16x^2-25}{16x^2-40x+25}$

31. $\frac{x^2-49}{x^2+5x-14}$

32. $\frac{-5x^2-2x+3}{5x^2+2x+3}$

33. $\frac{b^2-144}{b^2-24b+144}$

34. $\frac{y^2+4y-21}{y^2-10y+21}$