Section 1.6
The Number Line

UNDERSTANDING NUMBERS

All the numbers that you have used so far in mathematics to compute operations and solve problems are called *real numbers*. A real number could be any number, fraction or decimal.

Examples: The following are all real numbers:

$0.\overline{1}$ $\frac{1}{2}$ $\sqrt{5}$ π 6.524 40 10,000

Real numbers may be broken down into *Irrational Numbers* and *Rational Numbers*.

Real Numbers → Irrational Numbers (non-repeating decimals like $\sqrt{5}$, π)

Real Numbers → Rational → Fractions and Repeating Decimals ($\frac{1}{2}$, $0.\overline{1}$, $4.\overline{325}$)

Rational → Integers (...–1, 0, 1...) → Whole (0, 1, 2...) → Natural (1, 2, 3...)

Irrational Numbers

Irrational numbers are those numbers that cannot be written as a *ratio* of two integers.

Examples: The following are irrational numbers: $\sqrt{2}$ π $\sqrt{10}$ $\sqrt{500}$

Rational Numbers

A number is **ratio**nal if it can be written as a *ratio* of two integers, a fraction.

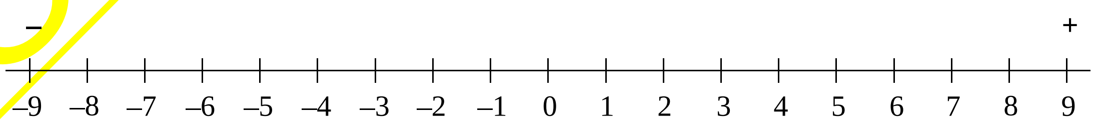
Examples: The following are rational numbers:

$0.\overline{3}$ $\frac{3}{4}$ $\frac{2}{1}$ $\frac{17}{4}$ $8.\overline{4532}$ 10.3 $\sqrt{169}$

Repeating decimals and square roots of *perfect squares* are also rational numbers.

UNDERSTANDING INTEGERS

Rational numbers not written as a ratio are called *Integers*. Integers can be positive or negative and they include *zero*. Integers can be best understood by the use of the *Number Line*.



Number lines have a *zero* in the middle, *negative* numbers to the left, and *positive* numbers to the right. To use the number line to find answers, start at zero and then move left and right according to whether the expression represents an addition or subtraction. Where the last operation lands, it gives the answer.

Example: Evaluate $-3 + 5 + 7 - 9 + 4 - 6 + 2 + 8 - 15$

Starting at zero move 3 to the left (because it is negative 3), 5 to the right, 7 right, 9 left, 4 right, 6 left, 2 right, 8 right, 15 left = –7

Adding Integers
Using the number line and adding positive integers gives a positive answer: $45 + 15 = 60$

Using the number line and adding negative integers gives a negative answer. $-20 + (-13) = -33$

Using the number line and adding a negative and a positive number gives an answer that could be either positive or negative, depending on whether the larger number is positive or negative:

$$+14 + (-8) = 6 \qquad +12 + (-19) = -7$$
$$14 - 8 = 6 \qquad 12 - 19 = -7$$

Subtracting Integers
Subtracting integers is also called "algebraic subtraction", which involves "taking away."

Taking away a positive number: $15 - (8) = 7$ $\quad -15 - (8) = -23$
$15 - (+8) = 7$ $\quad -15 - (+8) = -23$

Examples:
Jerry has \$15 in his pockets and "takes away" \$8 to buy lunch: 15 – (8) = \$7 left.

Jerry uses his credit card (he owes) to spend \$15 for groceries and \$8 for lunch:

–15 – (8) = –23 (owes \$23)

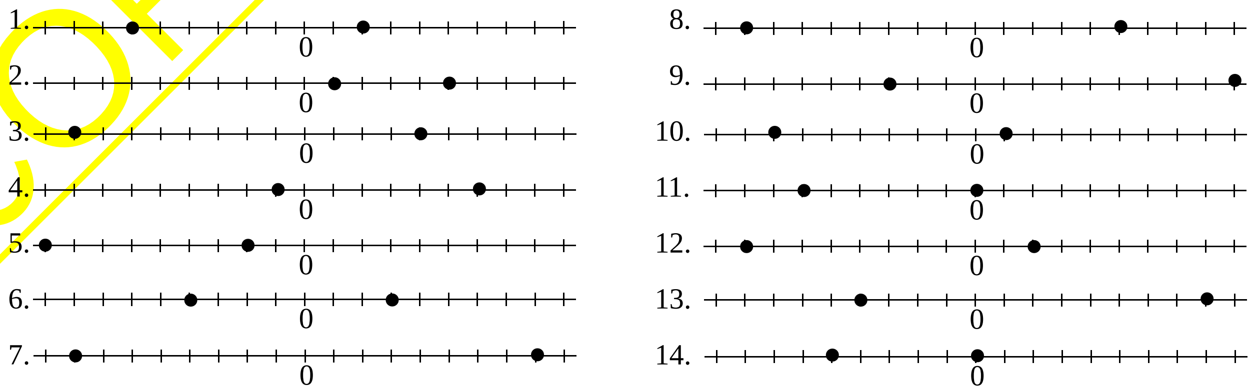
Taking away a negative number: $25 - (-10) = 35$ $\quad -25 - (-10) = -15$
$25 + 10 = 35$ $\quad -25 + 10 = -15$

Examples:
Jerry has \$25 left after paying \$10. If the payment is returned: 25 – (–10) = \$35

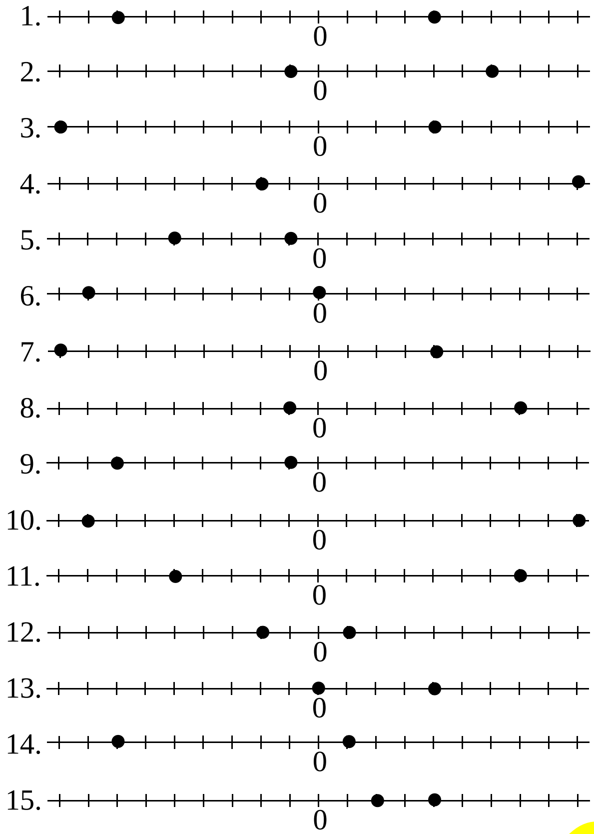
On credit, Jerry buy \$25 in groceries. If \$10 are returned: –25 – (–10) = –15

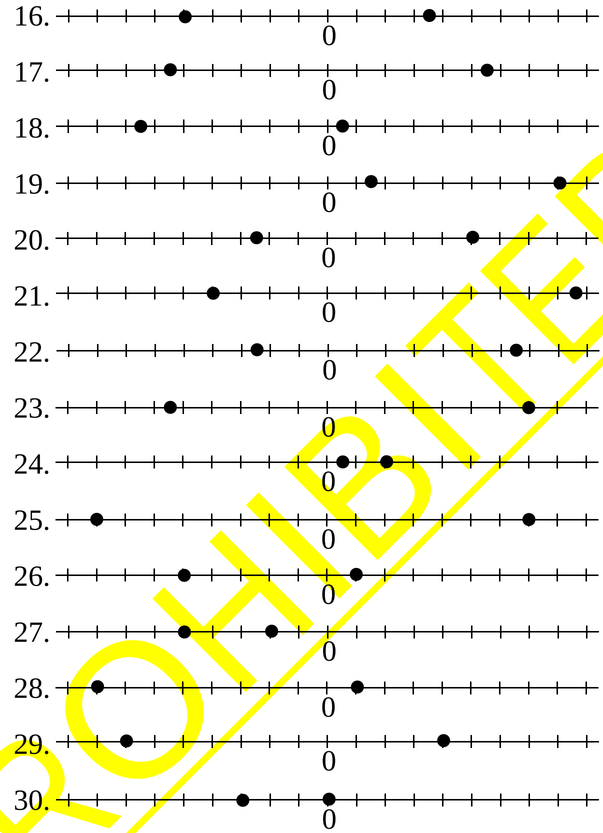
Because there is a benefit from taking away a negative, the product of negative × negative is positive.

Practice:
Identify both points and find the distance between them. Each tick mark represents a unit.



Find half the distance of the given set of points.





Subtract the lowest point from the highest point.