

Section 1.5

Ratios, Proportion, and Percent

RATIOS AND PROPORTION

A **ratio**, also called a fraction, is a comparison between two numbers using division.

Example: A car is travelling at 55 miles PER hour.

This is a ratio expressed in $\frac{\text{miles}}{\text{hour}}$ (the word PER takes the place of the fraction line)

Example: The price of rice is \$47 for 25 pounds. $\frac{\text{DOLLARS}}{\text{POUND}} = \frac{47}{25}$

Example: The pitcher is allowing 2.75 runs per 9 innings. $\frac{\text{RUNS}}{9 \text{ INNINGS}} = \frac{2.75}{9}$

A **proportion** is an arithmetic sentence stating that *two ratios are equal*. $\frac{a}{b} = \frac{c}{d}$

Because proportions contain four numbers, proportions are used to solve ratio problems where one of the four numbers is missing. Any of these four combinations are solved using cross multiplication and division, in that order. So that:

$$\frac{a}{b} = \frac{c}{d} \text{ is } ad = bc \text{ and } a = \frac{bc}{d} \quad b = \frac{ad}{c} \quad c = \frac{ad}{b} \quad d = \frac{bc}{a}$$

Simply stated: To find any **unknown** in a proportion, multiply across and divide by the third number. In the case of (a) above, multiply (b)(c) and divide the result by (d); for (b), multiply (a)(d) and divide by (c); for (c), multiply (a)(d) and divide by (b); for (d), multiply (b)(c) and divide by (a).

Example: A person in an automobile travels 260 miles in 4.5 hours. At this rate, how far can she travel in a period of 9 hours?

She travels in miles per hour $\frac{\text{miles}}{\text{hour}}$ or $\frac{260}{4.5}$ or how many miles in nine hours $\frac{\text{miles}}{9}$

$$\frac{260}{4.5} = \frac{m}{9}$$

Multiply across (260)(9) = 2340. Divide the results by 4.5. $\frac{2340}{4.5} = 520$ miles

To check, include the answer in the proportion $\frac{260}{4.5} = \frac{520}{9}$ and cross multiply $2340 = 2340$

Example: A soldier brings home a recipe that requires 60 pounds of butter to make 1,100 cookies. How many pounds of butter will he need at home if he only wants to make 40 cookies?

The units involved are pounds of butter and number of cookies, or how much **butter** per **cookie**: $\frac{B}{C}$

$$\frac{B}{C} = \frac{60}{1100} = \frac{B}{40}$$

Multiply and divide: $B = \frac{60 \times 40}{1100} = \frac{2400}{1100} = 2.18$ pounds of butter.

PERCENT

Percent problems are also proportional problems, where a percent ratio $\left(\frac{x}{100}\right)$ is always equal to a known ratio.

Example: A shirt advertised for \$40 is reduced \$8 and you buy it for \$32. What percent of the original price did you SAVE?

$$\frac{SAVE}{PRICE} = \frac{8}{40} = \frac{S}{100}$$

Multiply and divide: $\frac{8 \times 100}{40} = \frac{800}{40} = 20\%$ Answer: 20% savings

Example: If the sales tax rate in a certain state is 7%, how much sales tax must you pay for an item that costs \$45?

$$\frac{TAX}{COST} = \frac{T}{45} = \frac{7}{100}$$

Multiply and divide: $\frac{45 \times 7}{100} = \frac{315}{100} = 3.15$ Answer: \$3.15

Example: A certain bank requires a 15% down-payment for a house. If you have \$63,750 for a down-payment, what is the highest price you could pay for a house?

$$\frac{DOWN \ PAYMENT}{PRICE} = \frac{63750}{P} = \frac{15}{100}$$

$\frac{63750 \times 100}{15} = \frac{6375000}{15} = 425,000$ The price cannot exceed \$425,000.

Practice:

1. Jack's boat consumes 1.8 gallons of fuel per mile while his brother's boat consumes 2.7 gallons per mile. What is the ratio in fuel consumption of Jack's brother's boat to Jack's boat?
2. Printed checks cost \$8.40 per pack of 125 checks. How much is the total price for an order of 375 checks?
3. A company that rebuilds old generators for automobiles bought 60,000 diodes for \$3,000. If diodes come 75 to a box, how much is one box worth?
4. A case of blue shirts sold for \$3,000 and a case of white shirts for \$2,500. What is the ratio of the value of the blue shirts to the TOTAL value of the shirts?
5. The members of a math class received the following grades: 7 As, 13 Bs, 18 Cs, 6 Ds, and 2 Fs. What is the ratio of Cs to Fs?
6. In problem 5, what is the ratio of As to the total number of students?
7. Mario used 3.5 pounds of flour to make 56 cupcakes. How many pounds of flour will he need for 6,400 cupcakes?
8. Chris earned a \$2,800 commission on a house she sold for \$70,000. At this rate, how much will she earn on a house valued at \$240,000?
9. How much is the sales tax for a brand new \$65,000 car if the sales tax rate is 7.5%?
10. In a complaint to the city commission, Ms. Grant claimed that 8,000 "feral cats" were depleting the birds in a sanctuary nearby. If the city has 25,000 cats, what percent are feral?
11. If a state has a sales tax rate of 8.5%, how much will you pay for a \$2,500 computer?
12. A hotel charges \$120 per night for a room and, additionally, you must pay a 7% state sales tax and a city 5% bed tax. If you stay 5 days in the hotel, how much is the total bill for the 5-night stay?
13. A pair of shoes Yasmin wants to buy sells for \$75. If the store is having a 45% OFF sale, how much will she save on the pair of shoes?
14. In building a boat Marvin spends \$2,310 for materials, \$3,780 for labor and \$487.20 for different sales taxes. What percent of the money was spent in sales taxes?
15. Myrna bought a food mixer that originally sold for \$90. If she bought it at 30% off, how much did she pay for the mixer?
16. Betty sold 1,100 of the tickets sold for the basketball game. If the game was sold-out and there are 2,000 seats in the gym, what percent of the tickets did Betty sell?
17. Salvatore bought a radio and a car alarm. He paid a 5% sales tax. If the price of the radio was \$150 and the cost of the alarm was \$110, how much was his total bill?
18. A credit union pays 6.5% per year on all certificates of deposit. If Lisa has two certificates of \$10,000 each, how much interest will she receive after one year?
19. Buddy bought a cat at a "cat clearance sale." If the sale was for 25% off and the original price of the cat was \$250, how much did he pay for the cat?
20. The Big-Five Club collected donations for the homeless. If they spent \$0.52 of every dollar for food, how much of a \$25 donation went for food?
21. The gift you bought through the internet has a 7% handling charge. If your total payment was \$18.19, how much was the gift?
22. At the end of the year, Misty deposited \$5,000 in a bank to buy Christmas presents the following year with the interest earned. If the bank pays 8% interest per year, how much will she have for presents?

23. While buying a car, Maggie was offered different discounts and incentives totaling 28%. If the car she picked is listed for \$22,000, how much will she pay for the car?
24. A bread Paul is baking increases 35% from the time he puts it in the oven to the time that he removes it. If the final volume of the bread is 1,620 cubic inches, what was the original volume of the bread before going into the oven?
25. A concrete mix requires 3 bags of cement for every 5 bags of sand. If the job you are doing calls for 360 bags of cement, how many bags of sand are needed?
26. Javier's new salary is \$42,560 per year. If his old salary was \$38,000, by what percent was his salary increased?
27. Geraldine's medical education is estimated to cost \$85,000 per year. By paying for it in advance, she will save 15% of the cost. Find how much she will save in her three years of medical school.
28. A cruise ship uses 0.4 tons of fuel every 12 hours. Find the fuel consumption of the ship for a vacation tour that last 2 weeks.
29. A flat screen monitor that retails for \$800 is now being offered for \$320. Find the percent discount of the offer.
30. After reducing safety requirements by 17%, Valerie is re-designing a bridge with a yield strength of 175,000 kilograms. If the Chief Engineer brings safety requirements back up, find the original yield strength for the design of the bridge. Round off answer to the nearest hundred.
31. A recipe for candy uses 75 pounds of corn syrup to make 3,000 pieces of candy. If you want to reproduce the recipe for 50 candy pieces, how much corn syrup will you need?
32. Find the percent of the decrease if the price for porkbelly futures in the commodities market of Chicago dropped from \$27.00 to \$25.38.
33. Thirty-eight percent of the chickens in Cindy's farm lay large-grade eggs. If there are 52,000 chickens at her farm, how many do not lay large-grade eggs?
34. The price Jasmine paid for a wedding dress had been reduced 30%. If she paid \$1015.00 for it, what was the original price of the dress?
35. The track at the Indianapolis 500 Motor Speedway is an oval 2.5 miles long. How many seconds does it take for a racecar to run once around the track when the vehicle travels at 225 miles per hour?
36. The amount of water used per day by an average household increased from 58.5 gallons in 1950 to 135.72 gallons in 2000. Find the percent of the increase.
37. The bed tax at a Caribbean resort is 11%. If you stay a week at \$150 per night, how much is your total bill, including tax?
38. The depth of a lake in Canada increases 22% after the summer months. If the new depth is 151.28 feet, how deep was the lake before the summer?
39. A 4-inch wide by 6-inch high photograph is to be enlarged and fit in a space 54 inches high. How wide must the photograph be to keep all its proportions?
40. A skirt 38 inches long gets wet and shrinks to a length of only 34 inches. What percent of the length did the dress lose? Round off answer to the nearest tenth.
41. Find the interest earned after one year if \$34,000 is deposited in a bank account that earns 5.2% per year?
42. One single stalk of sugar cane in the Dominican Republic yields approximately 8 tablespoons of sugar, while a similar stalk in Florida yields 5. What is the percent increase between the two?
43. Find the total price of a \$45 shirt if the sales tax rate is 8.5%.