Section 1.4
The Properties of Algebra

COMMUTATIVE PROPERTY

Have you ever noticed that we CAN add any number in any order we want, but we CAN'T do the same with subtraction? Also, we can multiply in any order we want, but not in division.

Example:

We can add $3 + 5 + 2 = 10$
and switch the order to $5 + 3 + 2$
and still get the same answer.

Or multiply $8 \times 6 \times 4 = 192$
and re-arrange to $6 \times 4 \times 8$
and still get the same answer.

If you try the same with subtraction and division, it will not work. This is what is called the **commutative** property of addition and multiplication: Changing the order of the numbers will not affect the answer. The word *commutative* means exchange.

ASSOCIATIVE PROPERTY

Another neat truth about addition and multiplication is that you can "**associate**" as many numbers as you want and still get the same answer. This association is done by the use of parentheses.

Example:

$(12 + 8) + 14 = 12 + (8 + 14)$
Whether you add 12 and 8 first, or 8 and 14 first, the answer will not change.

$7(18 \times 2) = (7 \times 18)\ 2$
Whether you multiply 18 and 2 first, or 7 and 18 first, the answer will not change.

DISTRIBUTIVE PROPERTY

The example that follows tells you that you can add first and then multiply, or you can multiply first and then add. It doesn't matter; the answer will always be the same. Because the number outside the parenthesis is distributed over all the numbers inside the parenthesis, we call this a "**distributive**" property.

Example:

add first, then multiply
$3(9 + 4 + 6) = 3\ (19) = 57$

multiply first, then add
$3(9 + 4 + 6) = 3(9) + 3(4) + 3(6) = 27 + 12 + 18 = 57$

Although according to "order of operations" rules you are supposed to do what is inside the parenthesis first, sometimes this is not possible. For example in

$$5(x + y + z) = 5x + 5y + 5z$$

x, y, and z cannot be added without knowing what x, y, and z represent. Therefore, "distribute" 5 over the THREE terms of the trinomial $(x + y + z)$

INVERSE PROPERTY

Inverse properties pair the arithmetic operations we use in mathematics. "Inversion" means that pairs of numbers do the opposite—they "undo" each other.

For example, when subtraction "undoes" addition, we call it the **additive inverse** (negative) property. In this case addition (+) and subtraction (–) are being paired.

When division "undoes" multiplication, it is called the **multiplicative inverse** (reciprocal) property. In this case multiplication (×) and division (÷) are being paired.

In algebra, we also pair the "square" (raising to the **second** power: $x \cdot x = x^2$, $3 \cdot 3 = 3^2 = 9$) with the "square root" ($\sqrt{9} = 3$). In other words, the *square* and the *square root* are opposite of each other.

Examples:

Addition—Subtraction $8 - 8 = 0$ $x - x = 0$

Multiplication—Division $5 \cdot \frac{1}{5} = \frac{1}{5} \cdot 5 = 1$ $x \cdot \frac{1}{x} = \frac{1}{x} \cdot x = 1$

Square—Square Root $4^2 = 16$ $\sqrt{16} = 4$ $x^2 = (x)(x)$ $\sqrt{x^2} = \sqrt{(x)(x)} = x$

IDENTITY PROPERTY

The identity property also has to do with addition and multiplication. If we add **zero** to any number, the answer is the number; if we multiply **one** by any number, the answer is the number.

The "**additive identity**" is: $3 + 0 = 3$ or $a + 0 = a$

And the "**multiplicative identity**" is: $7 \times 1 = 7$ or $a(1) = a$

Practice:

Find the algebraic property being represented.

1. $5(3 + 4 + 7) = 5(3) + 5(4) + 5(7)$
2. $19 \times 1 = 19$
3. $\sqrt{36} = 6$
4. $12(5) + 12(8) = 12(5 + 8)$
5. $1 \times 12 = 12$
6. $(5 + 83) + 4 = 5 + (83 + 4)$
7. $18 + 3 + 12 + 7 = 12 + 3 + 7 + 18$
8. $-7 + 7 = 0$
9. $13 \times 16 \times 41 \times 8 = 41 \times 16 \times 8 \times 13$
10. $43 + 0 = 43$
11. $(17 \times 18) \times 9 = 17 \times (18 \times 9)$
12. $12(2) + 12(9) + 12(10) = 12(2 + 9 + 10)$
13. $44 + 66 + 77 = 66 + 77 + 44$
14. $0 = 15 - 15$
15. $10 - 10 = 0$
16. $\frac{1}{12} \cdot 12 = 1$
17. $34 \times 1 = 34$
18. $9(13 + 14 + 17) = 9(13) + 9(14) + 9(17)$
19. $10 + 0 + 54 + 13 = 13 + 10 + 54 + 0$
20. $(22 + 13) + 21 = 22 + (13 + 21)$
21. $16 \times 43 \times 22 = 22 \times 16 \times 43$
22. $0 + 17 = 17$
23. $(15 \times 23) \times 41 = 15 \times (23 \times 41)$
24. $6(16) + 6(9) + 6(11) = 6(16 + 9 + 11)$
25. $23(2 + 5 + 88) = 23(2) + 23(5) + 23(88)$
26. $\sqrt{81} = 9$
27. $12 + 0 = 12$
28. $3(15) + 3(18) + 3(-33) = 3(15 + 18 - 33)$
29. $1 \times 32 = 32$