Section 1.3
Arithmetic and Geometric Sequences

A sequence is a series of numbers that follow a fixed pattern. A sequence can be arithmetic or geometric. Arithmetic sequences use the ***addition*** of a particular number; geometric sequences use the ***multiplication*** of a particular number. For example, the series 3, 6, 9, 12... is arithmetic and the series 3, 9, 27, 81 is geometric.

Finding a Missing Number in an Arithmetic Sequence
— First, determine if the sequence is increasing or decreasing.
— Secondly, find the difference between the first two numbers, the second and third number, the third and fourth number, and so on. If the difference is always the same (fixed), then there is a sequence. Continuing the sequence requires that the difference found is added (or subtracted) continuously.
Finding a Missing Number in a Geometric Sequence
— The multiplier by which the sequence is increasing (or decreasing) must be identified first. If the sequence increases (or decreases) by the same **factor**, then it is a geometric sequence. Continuing the sequence requires that each number be the product of the LAST number TIMES the factor by which the sequence is changing.

Example:
Find the missing number in the sequence. Determine if the sequence is arithmetic, geometric, or neither of the two.

4, 9, 14, ..., 24, 29...

The difference between the first and second number is 5 (9 – 4 = 5), between second and third is also 5 (14 – 9 = 5); therefore, the sequence is increasing by the same number, 5, and the sequence is arithmetic. The missing number is 19 (14 + 5 = 19). After 29 the next number is 34 (29 + 5 = 34), then 39, and so on.

Example:
Find the missing number in the sequence. Determine if the sequence is arithmetic, geometric or neither of the two.

...4, 7, 11, 16, ..., 29, 37...

The differences begin with three (7 – 3 = 4), then four (11 – 4 = 7), five (16 – 5 = 11)... therefore, the missing number is found by adding six to 16 to get 22 and seven to 22 to get 29, and so on. Because the next number is NOT found by adding the same FIXED amount, then the sequence is neither arithmetic nor geometric.

Example:
Find the missing number in the sequence. Determine if the sequence is arithmetic, geometric or neither of the two.

1, 3, 9, 27, ..., 243...

Because the differences do not fit an addition pattern (the differences are 2, 6, 18), the sequence is not arithmetic; however, the numbers do increase by a factor of 3 ($1 \times 3 = 3$, $3 \times 3 = 9$, $9 \times 3 = 27$); therefore, the missing number is 81 ($27 \times 3 = 81$) and the sequence is geometric.

Practice:
In the following exercises, determine if the sequence is arithmetic, geometric, or neither of the two. Then select the next two numbers that represent the sequence, if a sequence is found.

1. ...2, 4, 6, 8, ...
2. ...22, 19, 16, 13, ...
3. ...45, 53, 61, 69, ...
4. ...2, 6, 18, 54, ...
5. ...1.25, 2.5, 5, 10, ...
6. ...2, 4, 9, 16, ...
7. ...3, –1, –5, –9, ...
8. ...4, 8, 10, 20, ...
9. ...33, 44, 66, 77, ...
10. ...3, 4.5, 6.75, 10.125, ...
11. ...17, 1, –15, –31, ...
12. ...4, 12, 36, 108, ...
13. ...7, 2, –3, –8, ...
14. ...10, 12, 15, 19, ...
15. ...44, 42, 38, 36, ...
16. ...23, 26, 29, 32, ...
17. ...21, 2100, 210,000, 21,000,000
18. ...65, 45, 25, 5, ...
19. ...1, –4, 16, –64, ...
20. ...55, 48, 41, 34, ...
21. ...7, 21, 63, 189, ...
22. ...8, 6, 3, –1, ...
23. ...6, 7.2, 8.4, 9.6, ...
24. ...2, 7, 12, 17, ...
25. ...24, 24, 24, 24, ...
26. ...56, 54.5, 53, 51.5, ...
27. ...86, 77, 66, 57, ...
28. ...23, 14, 5, –4
29. ...7, 10.5, 15.75, 23.625, ...
30. ...89, 39, –11, –61, ...
31. ...22, 242, 2,662, 29,282, ...
32. ...33, 44, 55, 66, ...
33. ...43, 37, 27, 23, ...
34. ...13, 9, 5, 1, ...
35. ...54, 56.5, 59, 61.5, ...
36. ...4, –8, 16, –32, ...
37. ...3, 9, 27, 81, ...
38. ...34, 24, 14, 4, ...
39. ...11, 33, 55, 77, ...
40. ...66, 33, 3, –30, ...
41. ...2, –20, 200, –2,000, ...
42. ...9, 6, 3, 1, ...
43. ...10, 15, 22.5, 33.75, ...
44. ...55, 44, 33, 22, ...
45. ...43, 43, 86, 258, ...
46. ...12, 14, 16, 18, ...
47. ...32, 29, 26, 23, ...
48. ...145, 153, 161, 169, ...
49. ...4, 16, 64, 256, ...
50. ...1.2, 2.4, 4.8, 9.6, ...
51. ...20, 40, 120, 360, ...
52. ...13, –1, –15, –29, ...
53. ...2, 4, 5, 10, ...
54. ...11, 22, 44, 55, ...
55. ...3, 4, 6, 9, ...
56. ...7, 1, –5, –11, ...
57. ...5, 15, 45, 135, ...
58. ...17, 2, –13, –28, ...
59. ...1, 2, 5, 10, ...
60. ...3, 3, 3, 3, ...
61. ...230, 260, 290, 320, ...
62. ...0.051, 5.1, 510, 51,000
63. ...5.5, 4.5, 3.5, 2.5, ...
64. ...1, –5, 25, –125, ...
65. ...85, 76, 67, 58, ...
66. ...6, 18, 54, 162, ...
67. ...9, 5, 1, –3, ...
68. ...5, 7.5, 10, 12.5, ...
69. ...12, 17, 22, 27, ...
70. ...21, 21, 21, 21, ...
71. ...26, 24.5, 23, 21.5, ...
72. ...8.6, 7.7, 6.6, 5.7, ...
73. ...33, 24, 15, 6
74. ...9, 10, 15, 24, ...
75. ...9, 19, 9, 19, ...
76. ...2, 22, 242, 2,662, ...
77. ...3, 44, 96, 137, ...
78. ...4, 3, 2.5, 2.25, ...